SORTING OUT α and Γ for X-RAYS

Take a power-law in frequency $f_\nu(\nu) = f_{\nu_0}\nu^\alpha$ expressed usually in erg cm^{-2} s^{-1} Hz^{-1}. Here, f_{ν_0} is just a constant, defined to be the monochromatic flux at some reference frequency ν_0. Since $E = h\nu$, we can also reframe this as $f_E(E) = f_{E_0}E^\alpha$, expressed usually in erg cm^{-2} s^{-1} keV^{-1}.

The broadband flux between energies E_1 and E_2 in erg cm^{-2} s^{-1} is

$$F = \int_{E_1}^{E_2} f_E \; dE = \frac{E^{(1+\alpha)}}{(1+\alpha)}\Big]_{E_1}^{E_2} f_{E_0}$$

$$= \frac{E_2^{(1+\alpha)} - E_1^{(1+\alpha)}}{(1+\alpha)} \; f_{E_0} = \frac{E_2^{(1+\alpha)} - E_1^{(1+\alpha)}}{(1+\alpha)E^\alpha} \; f_E$$

So the monochromatic flux at any desired energy E in erg cm^{-2} s^{-1} keV^{-1} is

$$f_E = \frac{(1+\alpha)E^\alpha}{E_2^{(1+\alpha)} - E_1^{(1+\alpha)}} F$$

To convert to erg cm^{-2} s^{-1} Hz^{-1} use $\frac{1}{\mathrm{Hz}} = \frac{1}{\mathrm{keV}}\frac{\mathrm{keV}}{\mathrm{Hz}} = \frac{\mathrm{h}}{\mathrm{keV}}$ where $h = 4.138 \times 10^{-18}$ is Planck's constant in keV sec. Therefore, the monochromatic flux at any desired energy E in erg cm^{-2} s^{-1} Hz^{-1} is

$$f_\nu = \frac{h(1+\alpha)E^\alpha F}{E_2^{(1+\alpha)} - E_1^{(1+\alpha)}}$$

Now, the power law can also be expressed in terms of *photons* rather than energy units, that is

$$N_E(E) = N_{E_0}\frac{E^\alpha}{E} = N_{E_0}E^{(\alpha-1)}$$

This allows a popular but confusing redefinition of the photon *number index* Γ so that $N_E(E) = N_{E_0}E^{-\Gamma}$ whereby we see that since $\Gamma = (1-\alpha)$.

P.S. If further confusion is desired, in a standard X-ray definition, people unfortunately also use $f_E(E) = f_{E_0}E^{-\alpha_X}$