The Role of Statistics
in the Discovery of a Higgs Boson

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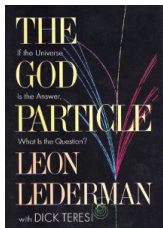January 2014

Outline

Outline

The "God Particle" in the Popular Press

Higgs and the holy grail of physics

By Lawrence M. Krauss, Special to CNN

July 6, 2012 -- Updated 1507 GMT (2307 HKT)

This boson is so central to the state of physics today, so crucial to our final understanding of the structure of matter, yet so elusive, that I have given it a nickname: the God Particle.

— Leon Lederman in "The God Particle" [1]

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[1] He goes on to say that the name "Goddamn Particle" might be a more appropriate, "given its villainous nature and the expense it is causing" (published in 1993).

Discovery of a Higgs Boson

The Higgs Boson

ARTICLE

Journey in the Search for the Higgs Boson: The ATLAS and CMS Experiments at the Large Hadron Collider
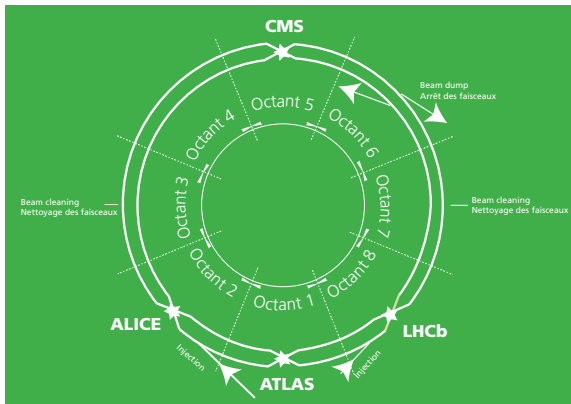
M. Della Negra,[1] P. Jenni,[2] T. S. Virdee[1]*

—— *Science*, 21 Dec 2012, Vol 338

- July 2012: Simultaneously announce discovery of "a new boson"
- P-values less than 1 in 3 million
- Behavior broadly matches predictions of Standard Models for the Higgs boson.
- March 2013: The new boson is promoted to "*a Higgs boson*"
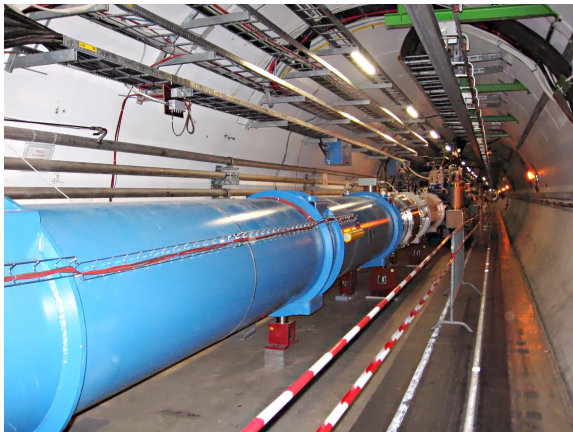
CERN's Large Hadron Collider

Proton-Proton Collisions:

LHC's 27km tunnel straddles the Swiss-French boarder.

The LHC Tunnel

Forbes: Finding The Higgs Boson Cost $13.25 Billion

What Does the Cost Mean for Statistical Analyses?

Researchers really don't want:

1. conflicting results from different analyses
2. change their minds about the choice of statistical methods
3. in any way be wishy-washy.

Committee decides the form of statistical analysis, and will

1. be conservative
2. be frequentist (subjectivity is bad!)
3. set out the analysis protocol in advance.

Probably the most careful frequentist analysis ever conducted.

A Simulation of Proton-Proton Collision
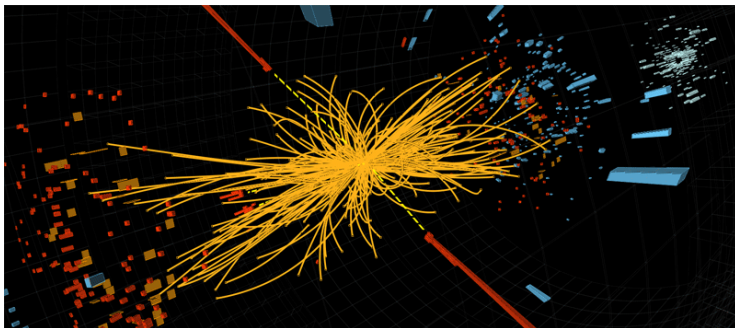
- Collisions produce a shower of new particles
- Unstable particles (e.g., Higgs) decay before detection
- Track trajectory, energy, and momentum to identify particles
- Compute *invariant mass* of particles formed in collisions

Decay Channels of the Higgs
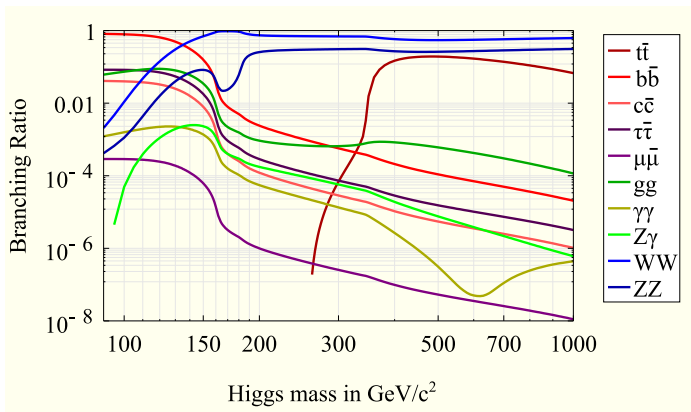
The Standard Model predicts the relative probabilities of Higgs decay channels as a function of its mass.

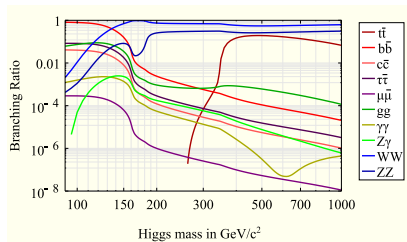# Decay Channels of the Higgs

Fig. 4 Feynman diagram illustrating the decay mechanism of a Higgs boson, H, to two photons, γ, via quantum loop process involving a W boson.
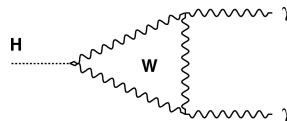
The ATLAS Collaboration Science 2012;336:1576-1582

Published by AAAS

- Primary Decay Channels for the Detection of the Higgs:
  - ★ Higgs into two photons ($H \rightarrow \gamma\gamma$)
  - ★ Higgs into two Z bosons ($H \rightarrow ZZ$), each decay into electrons or muons
- Not the most likely decay channels (together less than 1%).
- Advantageous signal-to-background ratio (Bayes Theorem)

Searching for the Bump above Background ($H \to \gamma\gamma$)
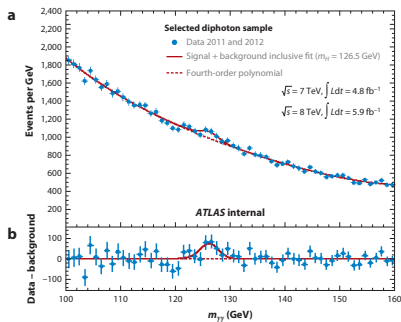
- Look at event counts in a number of invariant mass bins.
- Known physical processes result in background distribution
- Expect excess counts at invariant mass of Higgs boson.

Searching for the Bump above Background ($H \to \gamma\gamma$)
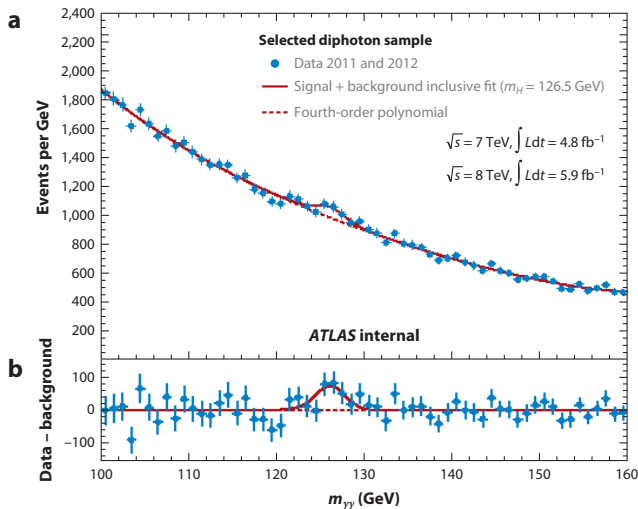
Possible Statistical Outcomes

1. **Discovery of new physical particle:** Conclude data are inconsistent with the null hypothesis of known background physics, but consistent with the hypothesized Higgs boson.

2. Conclude data inconsistent with Higgs boson hypothesis.

3. **Upper limit** on possible Higgs boson signal strength.

4. **Determine that experiment is not sensitive** enough to distinguish between new physics and background.

More sophisticated than accepting / rejecting H_0

Can we reject H_A?

Outline

A Simplified Model

- Data involve multiple invariant mass bins for each channel.
- To focus statistical issues we consider a single channel:

$$N_m \sim \text{POISSON}(\beta_m + \kappa_m \mu),$$

β_m is the expected background count in bin m
κ_m is the expected Higgs Boson count in bin m
μ is either one or zero.

- To hedge against misspecification, treat μ as continuous
- For the moment we assume the β_m and κ_m are known.

Detection and Upper Limits

Basic Model: $N_m \sim \text{POISSON}(\beta_m + \kappa_m \mu)$

Can we detect the Higgs? Conduct a hypothesis test that
compares $H_0 : \mu = 0$ with $H_A : \mu > 0$.

Upper Limit: In the absence of detection, compute an UL on μ.

- The hypothesis is tested with a Likelihood Ratio Test
- Standard asymptotics do not apply (H_0 on boundary)
- The test in inverted to compute intervals and upper limits
- *Unified Approach* for UL and intervals: maintains frequency coverage (Feldman & Cousins, 1998)

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Exclusion and Sensitivity

Basic Model: $N_m \sim \text{POISSON}(\beta_m + \kappa_m \mu)$

Can we exclude the Higgs? If the UL is too small to be
consistent with $\mu = 1$, exclude possibility of Higgs.

Compute Sensitivity: Is the hypothesis test powerful enough to
distinguish between background and Higgs?

Sensitivity is quantified by

- The median of the UL under H_0 or
- Smallest value of μ obtaining a given power for α-level test.

We would like both quantities to be small.

Treating the Higgs as the Null Hypothesis

We can formalize the relationship between detection, exclusion, and sensitivity by comparing two hypothesis tests:

Standard Test: Compare $H_0 : \mu = 0$ with the hypothesis $\mu > 0$

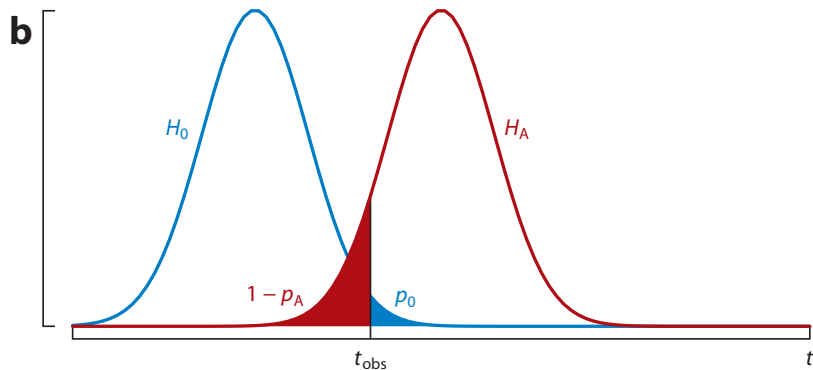$$p_0 = \Pr(T \geq t_{\mathrm{obs}} | H_0)$$

Reversed Test: Compare $H_A : \mu = 1$ with the hypothesis $\mu < 1$

$$1 - p_A = \Pr(T \leq t_{\mathrm{obs}} | H_A)$$

Small values of $1 - p_A$ are evidence for rejecting H_A and excluding the Higgs boson[2].
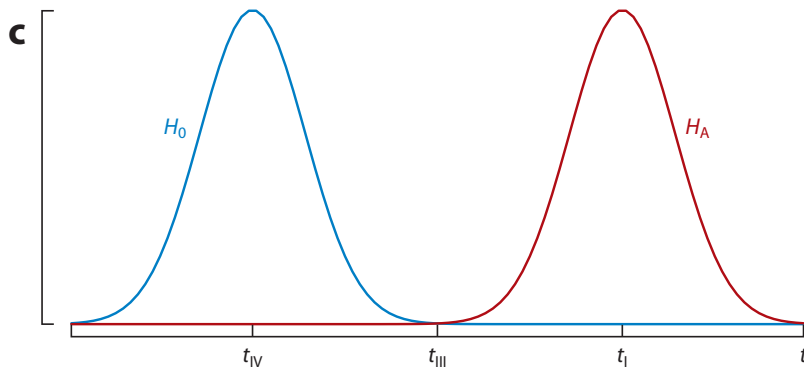
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[2]Here both p-values are defined to be left tail probabilities.

The Two P-values

A Sensitive Test

Not all values of t_{obs} lead to a clear decision.

Possible Outcomes

Four possible outcomes of the two tests:

	$p_0 \leq \alpha_0$	$p_0 > \alpha_0$
$1 - p_A > \alpha_A$	Case I: Detection	Case II: Both models are consistent w/ data
$1 - p_A \leq \alpha_A$	Case III: Neither model is consistent w/ data	Case IV: Exclusion

$\alpha_0 \ll \alpha_A$: Particle physicists are more concerned with false discovery than with missing a signal.

An Insensitive Test

Case II: Both models are consistent with data.
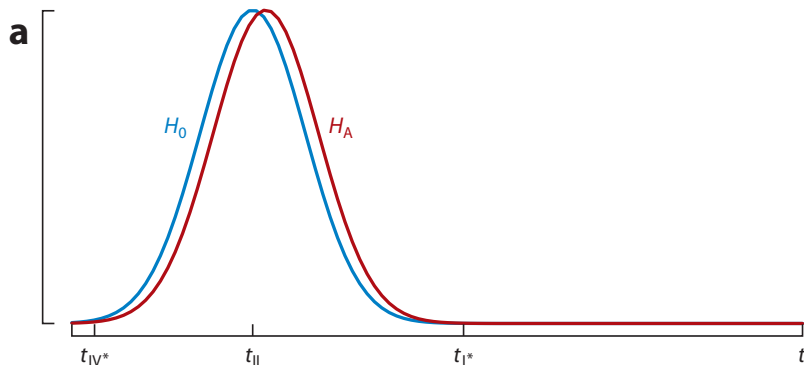
Two troublesome special cases: $I^\star$ and $IV^\star$

The Trouble with an Insensitive Test
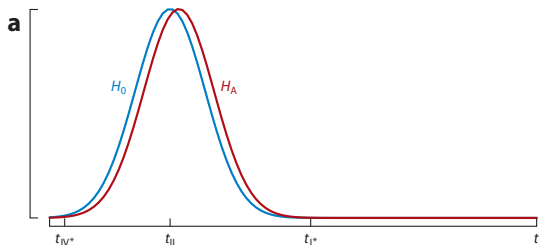
Case IV* : $1 - p_A$ is small, but data are unlikely under both models.

- Exclusion questionable: there are fewer counts than expected even without a Higgs boson.
- Can be caused by a downward fluctuation in background.
- Case I* is less problematic, because $\alpha_0 \ll \alpha_A$.

Comparing the Sensitivity of Tests (Lyons, 2013)

a
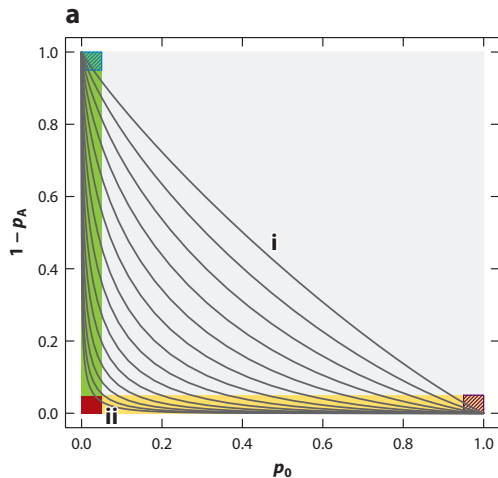
Green   Detection
Yellow Exclusion
   Red Neither Model
  Grey Both Models

   i insensitive test
   ii sensitive test

$\alpha_0 = \alpha_1 = 0.05$

Avoiding Exclusion Under an Insensitive Test

Case IV[*]
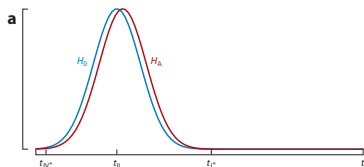
- Exclusion is unwarranted
- Do not exclude $\mu = 1$ if both $1 - p_0$ and $1 - p_A$ are small

Read (2000) suggested excluding $\mu = \mu_0$ only if

$$\mathrm{CL_S} \; = \; \frac{1 - p_A}{1 - p_0} \; = \; \frac{\Pr(T < t_{\mathrm{obs}} | \mu = \mu_0)}{\Pr(T < t_{\mathrm{obs}} | \mu = 0)} \; \leq \; \alpha_A.$$

$\mathrm{CL_S}$ ULs bound the values of μ that cannot be excluded.

Exclude μ if it makes $T < t_{\mathrm{obs}}$ much less likely than does $\mu = 0$

Effect of the $\mathrm{CL_S}$ Criterion
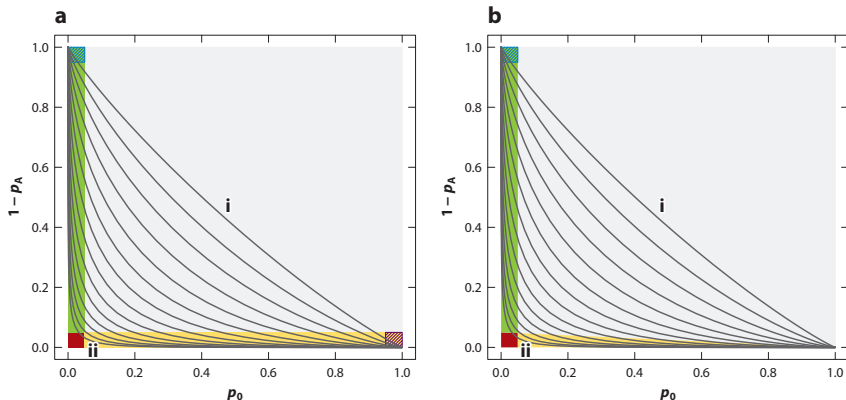
Troublesome purple region is almost eliminated with $\mathrm{CL_S}$.

5σ Detection Threshold

P-values are converted to SDs from 0, under std normal.

- P-value of 0.025 corresponds to a "1.96σ" result.
- July 2012 detections were 6σ and 5σ (ATLAS/CMS)

5σ is required for "discovery"

- High profile false discoveries led to conservative threshold
- Treat Higgs mass as known (multiple-testing)
- Calibration, systematic errors, and model misspecification
- Of course cranking down α_0 does not address these issues

*"In particle physics, this criterion has become a convention ...
but should not be interpreted literally[3]."*

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[3]Glossary in the *Science* review of the 2012 CMS and ATLAS discoveries.

Outline

Data Collection

- LHC produces millions of proton collisions per second, most are uninteresting
- Fast online *triggers* save only ~ 100 events per second
- As few as 1 in 10^8 stored events may involve Higgs boson
- Further *cuts* aim to prune data, reduce background, and focus analysis on potential new physics
- Cuts are based on values of individual variables and supervised learning algorithms trained on simulations and manually classified events

Counts in each Category that survive the triggers and cuts are used in analysis.

Data Processing

Within each channel, event counts are stratified into relatively homogeneous strata called *categories*.

- Categories with similar signal-to-noise increases power.
- Often based on *flavors* of decay, e.g.,
  - ⋆ $H \rightarrow ZZ$, 4 categories based on Z into electrons or muons.
- Sometimes based on boosted decision trees
  - ⋆ Aim to separate Higgs from background events.
  - ⋆ Use simulated data.
  - ⋆ Predictors: Momenta, energy, and presence of particles.
 - ⋆ Cut into categories based on the fitted probability of Higgs.

The search is based on the
Stratified Sample within each Channel.

Background Models

Parametric models are used for background events

- Owing to cuts and stratification, different background distribution in each category-channel pair

- Functional forms set by simulation with same cuts/strata

- Various background models are considered

- E.g., exponential of polynomials (order≤ 3), Bernstein polynomials (order≤ 7), etc.

- The fitted parameters are determined with real data

Fig. 5 Distribution of the invariant mass of lepton pairs for the zero-jet eμ category in the search at 8 TeV for the SM Higgs boson decay to a pair of W bosons.
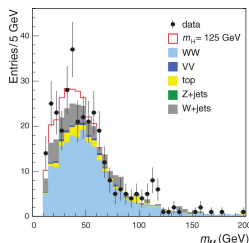
The CMS Collaboration Science 2012;338:1569-1575

Published by AAAS

Outline

Mass-by-Mass Analysis

Search conducted separately on fine grid of Higgs masses, m_H

- Integrated search may overlook masses w/ low sensitivity.
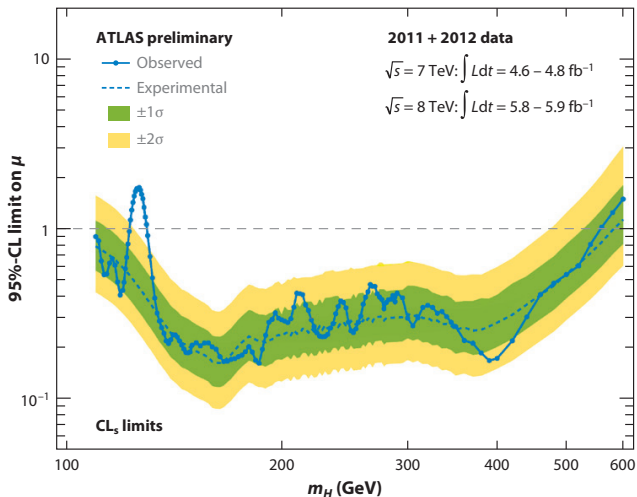- Mass-by-mass search allows sensitivity, ULs, and p-values to be computed for each potential mass.

Let m index mass bins, s categories (strata) , and c channels.
For a given Higgs mass, m_H,
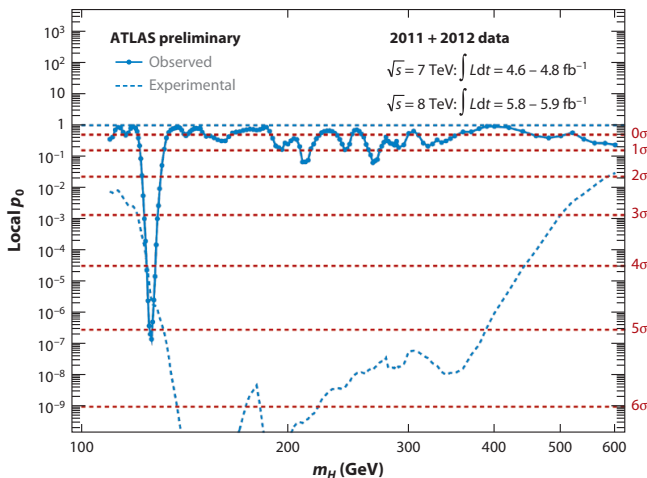
$$N_{msc} \sim \text{POISSON}\Big(\beta_{sc}(\theta_{sc}, m) + \kappa_{sc}(\phi_{sc}, m)\, \mu\Big).$$

- Models and their parameters vary among channel/categories
- Source model specifies the expected bump above background

$\mathrm{CL_S}$ Upper Limits on μ as a Function of m_H

Local P-values as a function of m_H

But what about Multiple Testing?

The Look Elsewhere Effect

- Let $T(m_H) \sim \chi_s^2$ be the LRT for the m_H-specific test.
- Davies (1987) shows

$$\Pr\left(\max_{m_H} T(m_H) > c\right) \leq \Pr\left(\chi_s^2 > c\right) + \mathrm{E}\left(M(c) \mid H_0\right),$$

- $M(c) =$ number of times $T(m_H)$ increases above c as $m_H \uparrow$
- To avoid MC evaluation of $\mathrm{E}(M(c)|H_0)$, let $c_0 \ll c$ and use[4]

$$\mathrm{E}\left(M(c) \mid H_0\right) = \mathrm{E}\left(M(c_0) \mid H_0\right) \left(\frac{c}{c_0}\right)^{(s-1)/2} \exp\left(-\frac{(c-c_0)}{2}\right),$$

- 6σ / 5σ significances reduce to 5.1σ /4.6σ (ATLAS/CMS)

[4]Gross and Vitells (2010)

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The Role of Statistics in the Discovery of a Higgs Boson

Estimating the Higgs Mass

- The Higgs mass is estimated via a unified analysis.
- Source model, κ_{sc} depends on m_H:

$$N_{msc} \sim \text{POISSON}\Big(\beta_{sc}(\theta_{sc}, m) + \kappa_{sc}(\phi_{sc}, m, m_H)\ \mu\Big).$$

Fig. 12 Confidence intervals comparing mass and signal strength for the H→γγ, H→ZZ→ℓℓℓℓ, and H→WW→ℓνℓν channels, including all systematic uncertainties.
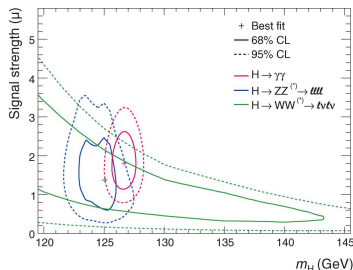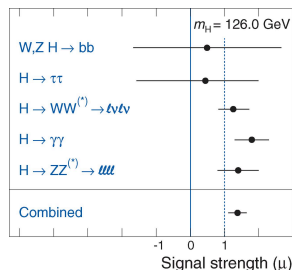
The ATLAS Collaboration Science 2012;338:1576-1582

Fig. 13 Measurements of the signal strength parameter μ for mH = 126 GeV for the individual channels and their combination.

The ATLAS Collaboration Science 2012;338:1576-1582

Outline

Is a coherent analysis of the full model possible?

$$N_{msc} \sim \text{POISSON}\Big(\beta_{sc}(\theta_{sc}, m) + \kappa_{sc}(\phi_{sc}, m, m_{\text{H}})\,\mu\Big).$$

Consider the LRT for $\mu = 0$ vs $\mu > 0$:

1. The null space is on the boundary.
2. Worse, m_H is unidentified under H_0.
3. Still worse, a sharp null: p-value can vastly overstate evidence for alternative[5], Jeffrey-Lindley paradox.

How about Bayes Factors:

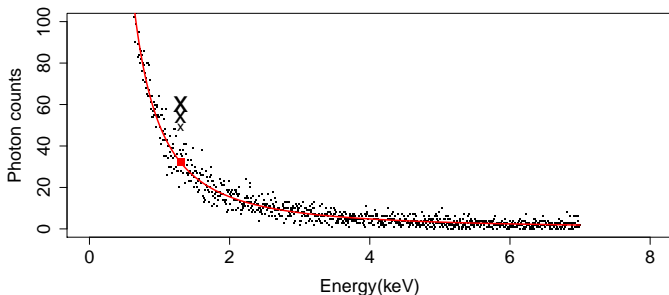1. Highly dependent on choice of prior for μ and m_H.

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[5] E.g., Berger & Delampady, *Testing Precise Hypotheses*, Stat. Sci., 1987

A Similar Example in High-Energy Astrophysics

$$Y_i \overset{\text{ind}}{\sim} \text{POISSON}(\alpha E_i^{-\beta} + \omega I_{\{i=\mu\}})$$

Is there sufficient evidence to conclude that $\omega > 0$??

- Fix α and β throughout
- The "true" emission line is at $\mu = \textbf{1.3}$ keV.

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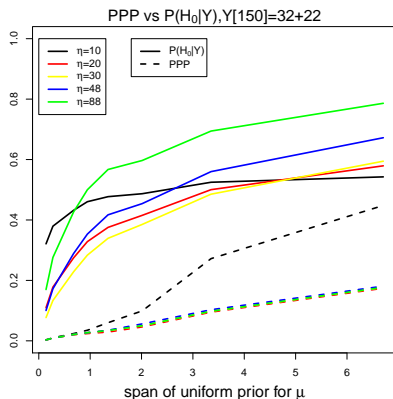# Results: Comparing Bayes Factors with P-values

Prior on spectral line: $\omega \sim U(0, \eta)$ *and* $\mu \sim U(1.3 \pm \kappa)$.

$P(H_0|Y)$ under 50-50 prior.

P-values vastly overstate evidence for line.

Prior on μ
- *let's us decide where to look.*
- *penalty for many looks.*
- *i.e., look elsewhere effect.*
- *Sensitivity of Bayes Factor to prior for μ is sensible.*
- *Prior on line intensity: look for weak or strong lines.*

PPP vs P(H0|Y),Y[150]=32+22

Legend:
- $\eta=10$
- $\eta=20$
- $\eta=30$
- $\eta=48$
- $\eta=88$
- P(H0|Y)
- PPP

span of uniform prior for μ

Conclusion

- Remember: all real data analyses involve trade-offs.
- Higgs researchers have insisted on much higher standards than are typically encountered in practice.
- They should be commended for the caliber of their methods and their discovery recognized as an excellent example of the dynamic interplay of modern statistical methods with a complex real-world applied problem.

Further Reading:

- van Dyk, D. A. (2014). The Role of Statistics in the Discovery of a Higgs Boson. *Annual Review of Statistics and Its Application*.
- December 2012 Special Issue of *Science*
- PhyStat Conference Proceedings Series