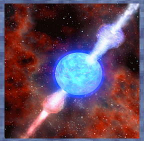
Quantifying the Non-Existent

Multi-wavelength Model Fitting with Non-Detects

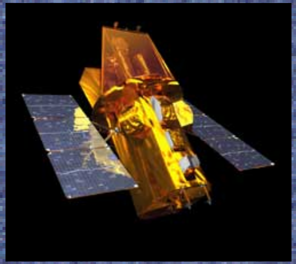
Tanmoy Laskar
Ph.D. Student in Astronomy
Harvard University

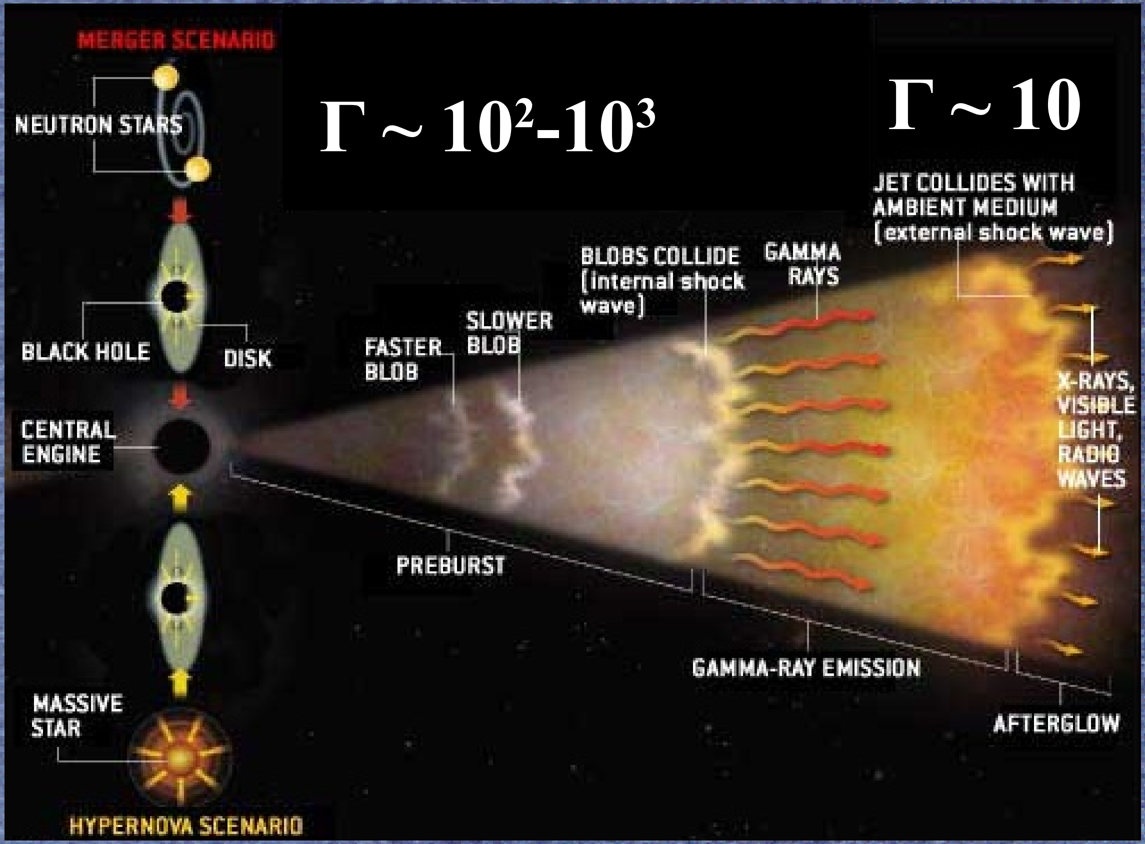
Outline

- Context – Gamma Ray Bursts and their Afterglows
  - The Phenomenon
  - Light Curves and Modeling
- Radio Interferometry
  - Visibilites and The van Cittert – Zernike Theorem
  - Deconvolution
  - The Measurement Process
- Incorporating Non-detects

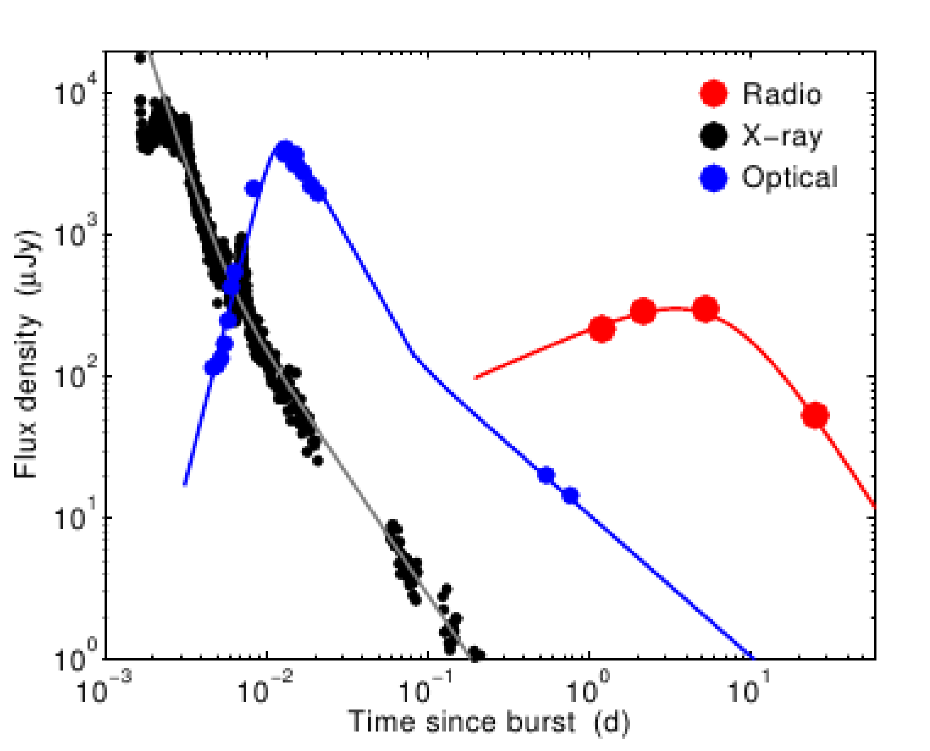


Gamma Ray Bursts





GRB Lightcurves

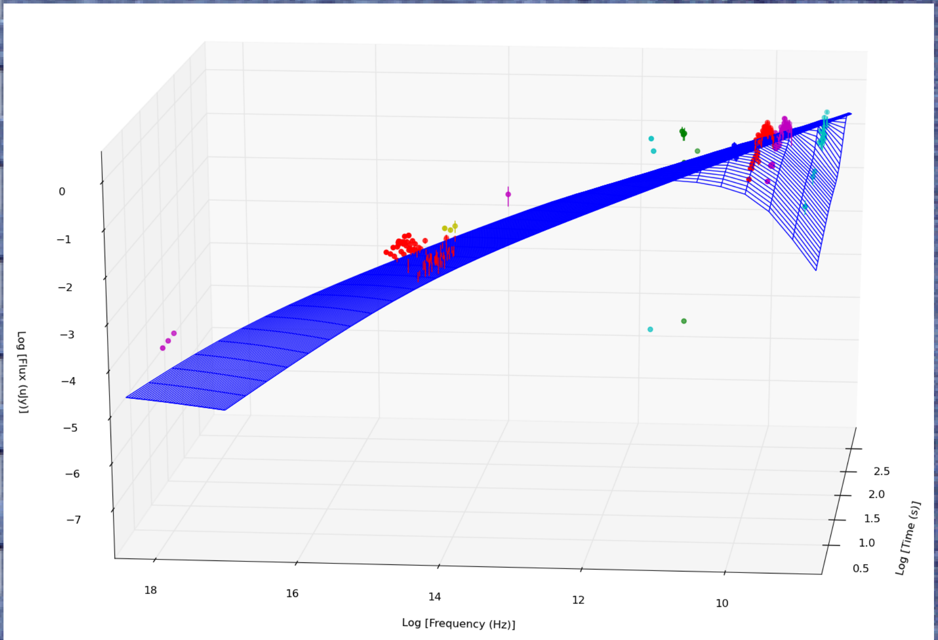


Laskar et al. (in prep.)

$$M(\vec{P}) = F_M(\nu, t)$$

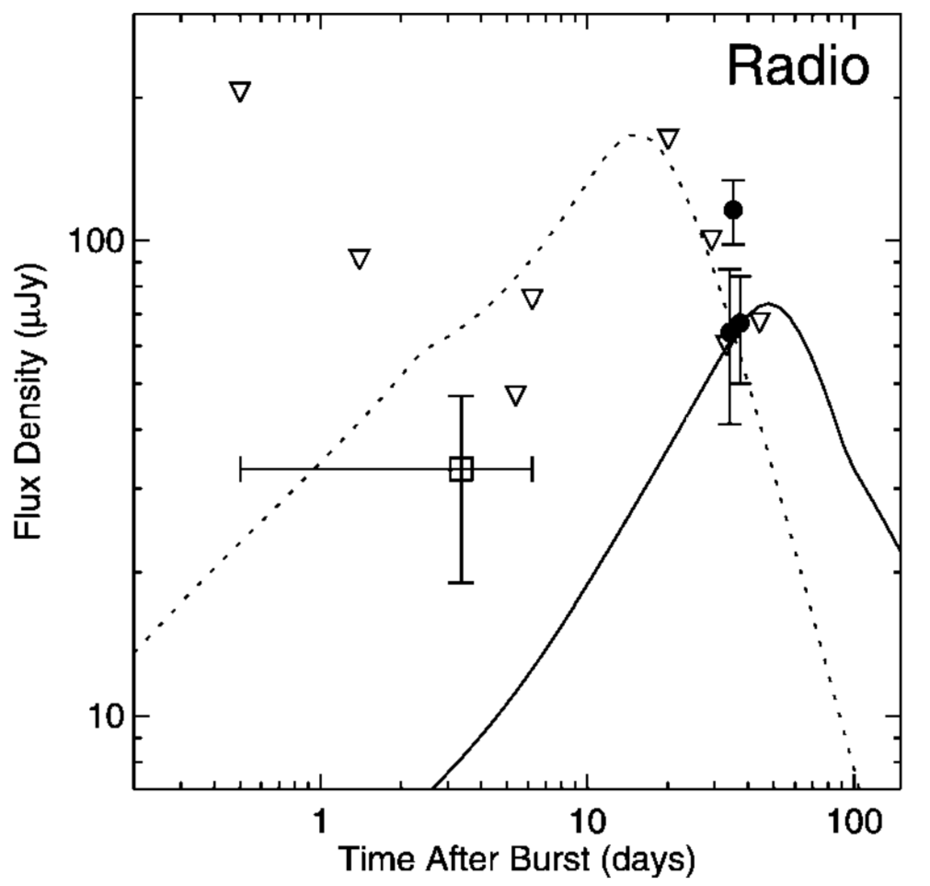
- Light curve = $F_\nu(t)$
- Afterglow first appears in X-rays
- Peak emission moves to lower frequencies with time

Multi-wavelength modeling



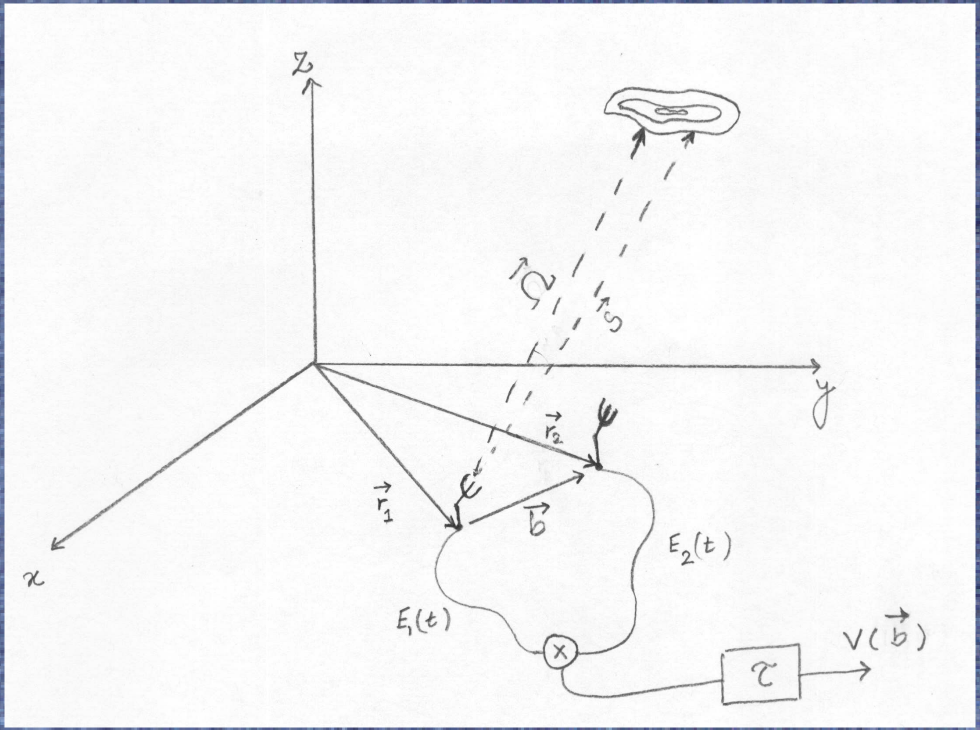
Laskar et al. (in prep.)

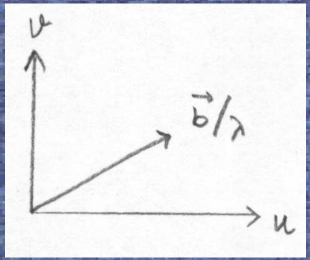
$$M(\vec{P}) = F_M(\nu, t)$$



Frail et al. (2006)

Radio Interferometry





$$V(u,v)=\lim_{T\to\infty}\frac{1}{T}\int E_1(t)E_2(t)\,dt$$

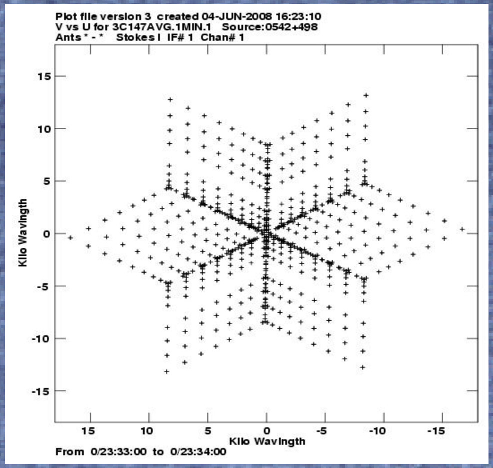
$$V(u,v)=\int I(\alpha,\beta)e^{2\pi i(u\alpha+v\beta)}\,d\alpha\,d\beta$$

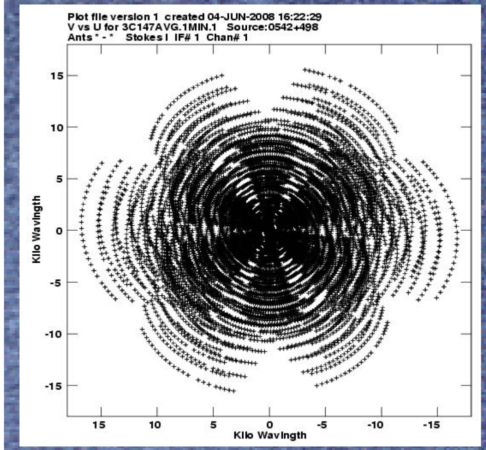
Measuring V(u,v)

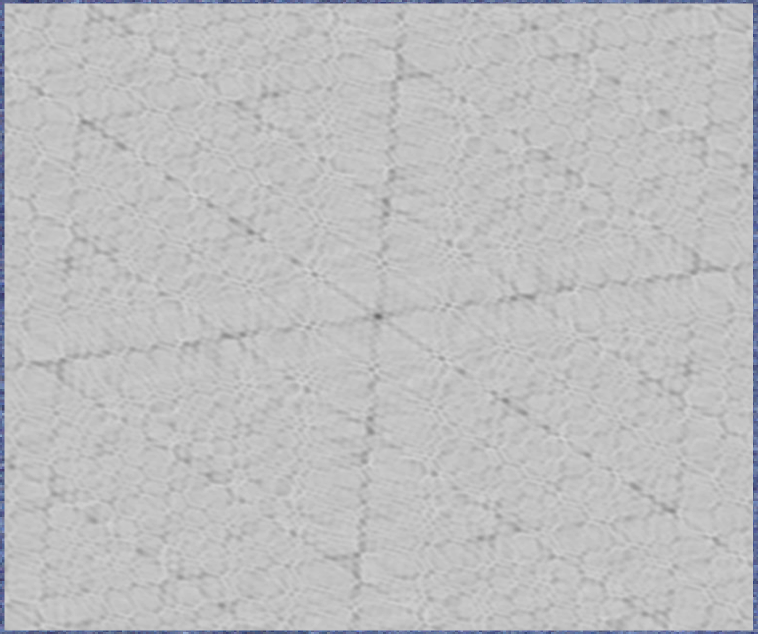
$$V_m(u,v) = W(u,v)\,V(u,v)$$

$$IFT(V_m(u,v)) = IFT(W) * IFT(V)$$

$$= B(\alpha,\beta) * I(\alpha,\beta)$$







Deconvolution

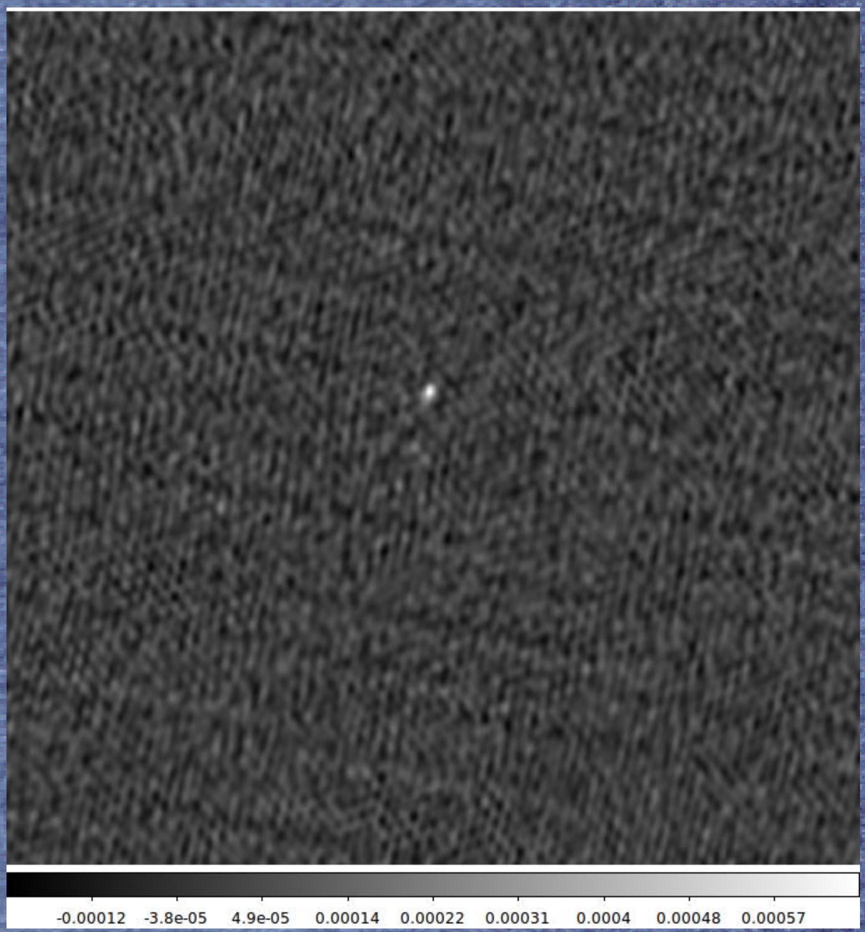
The CLEAN Algorithm:

- Find map max
- Subtract (gain x dirty beam at position of peak) from map
- Add δ-function to clean components list
- Repeat

Hogbom (1974)

1. "Dirty image" = IFT V_m
2. Clean: obtain clean components and residuals
3. Restore: convolve cc with "clean beam" and add back residuals

Detection?



- Fit the source with the known clean beam (elliptical Gaussian)
- Obtain source parameters (shape, size), both convolved with and deconvolved from clean beam
- Non-detect? Report upper limit = 3 x rms of map …

Model Fitting – Maximum Likelihood

$$t_i = min(x_i, c_i)$$

x_i : detected values

c_i : upper limits

$$\delta_i = \begin{cases} 0 \text{ (censored)} \\ 1 \text{ (detected)} \end{cases}$$

$$L = \prod_{i=1}^{N} P[t_i, \delta_i] = \prod_{i=1}^{N} [f(t_i)]^{\delta_i} [1 - S(t_i)]^{1-\delta_i}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}} \qquad S(x) = 1 - \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right]$$