Astronomical source detection and background separation via hierarchical Bayesian nonparametric mixtures

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Collaborators

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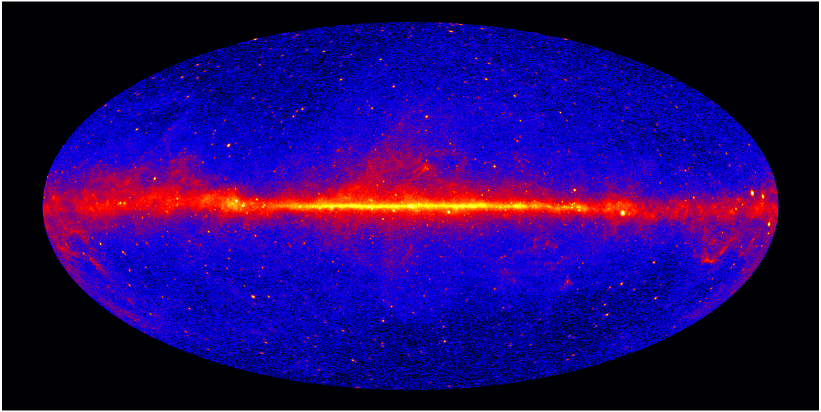
- David Stenning

High-energy astronomical count maps



(Image Credit: NASA/DOE/Fermi LAT Collaboration)

High-energy astronomical count maps

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  $\mathcal{D}_{ijk} = \#$photons spatially projected into ij with energy level k.

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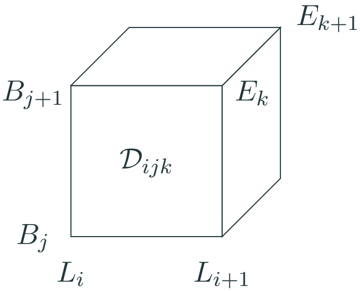

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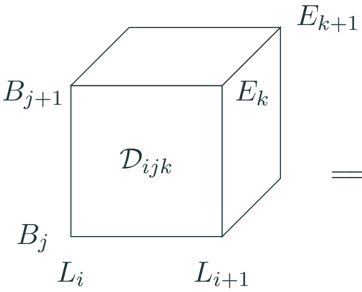

$\Longrightarrow$

i	L_DEG	B_DEG	Energy
1	$\frac{L_i+L_{i+1}}{2}$	$\frac{B_j+B_{j+1}}{2}$	$\frac{E_k+E_{k+1}}{2}$
2	$\frac{L_i+L_{i+1}}{2}$	$\frac{B_j+B_{j+1}}{2}$	$\frac{E_k+E_{k+1}}{2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathcal{D}_{ijk}$	$\frac{L_i+L_{i+1}}{2}$	$\frac{B_j+B_{j+1}}{2}$	$\frac{E_k+E_{k+1}}{2}$

The astronomical maps

The statistical unit: Photons $i = 1, \dots, n$ with directions

$$\mathbf{x}_i = (x_i, y_i) \quad \in \quad \mathcal{X} = (x_{min}, x_{max}) \times (y_{min}, y_{max})$$

and energy level

$$E_i \in (E_{min},\ E_{max}).$$

Two main relevant sources of information:

- Astronomical sources at different levels of energies;
- background contamination.

Main Goals

Start from $\{(x_i, y_i, E_i)\}_{i=1}^{n}$

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1. **discover** and locate **the high-energy astronomical sources** in a sky map;
2. **quantify** their **intensities**;
3. **distinguish** them from **the irregular background contamination** spread over the analysed area.

A very general statistical model

- $\mathbf{x}_i = (x_i, y_i), E_i$ with $i = 1, \ldots, n$ and $Z \in \{0, 1\}$,

$$\mathbf{x}_i, E_i | Z_i = z \sim \begin{cases} s(\mathbf{x}_i, E_i) & z = 1 \implies \text{Source model} \\ b(\mathbf{x}_i, E_i) & z = 0 \implies \text{Background model.} \end{cases}$$

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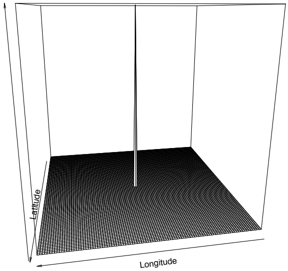
- Given $\delta \in (0, 1)$,

$$f(\mathbf{x}_i, E_i) = \delta s(\mathbf{x}_i, E_i) + (1 - \delta) b(\mathbf{x}_i, E_i).$$

Single source model



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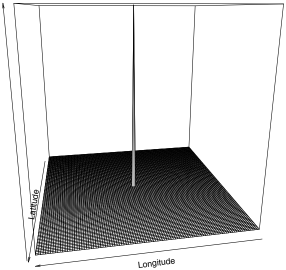

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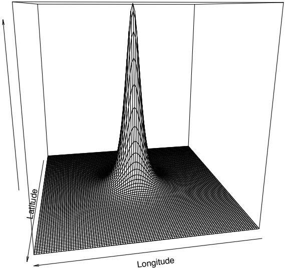

- A photon i from a source j spreads around it as

$$\mathbf{x}_i|E_i \sim \mathcal{N}(\boldsymbol{\mu}_j, \sigma^2_{E_i} I_2), \qquad \boldsymbol{\mu}_j \in \mathcal{X}.$$

- The spectral information from the sources is taken as

$$E_i \sim Par_t(\lambda_s, E_{\min}, E_{\max}), \qquad \lambda_s \in (1, 4).$$

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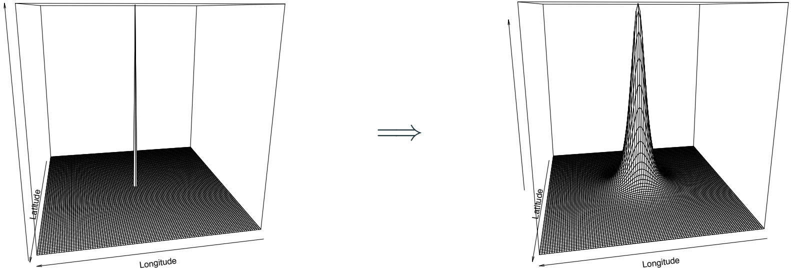

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Unknown Parameters	***Known Parameters***
$\boldsymbol{\mu}_j$: location of the source j	$\sigma^2_{E_i}$: variance parameter
λ_s: spectral parameter	

Modelling $s(\cdot)$: some aspects to consider

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Multiple sources model

$$s(\mathbf{x}_i, E_i|\mathcal{F}, \lambda_s) = p(\mathbf{x}_i|E_i, \mathcal{F})g(E_i|\lambda_s),$$

where

$$p(\mathbf{x}_i|E_i, \mathcal{F}) = \int \phi(\mathbf{x}_i|\boldsymbol{\mu}, \sigma^2_{E_i} I_2)\mathcal{F}(d\boldsymbol{\mu}),$$

$$\mathcal{F} \sim \mathcal{DP}(\alpha_s, \mathcal{F}_0),$$

with

$$\mathcal{F}_0(\boldsymbol{\mu}) = \mathcal{U}(x_{min}, x_{max}) \times \mathcal{U}(y_{min}, y_{max})$$

.

Modelling $s(\cdot)$: an alternative representation

- The model can be rewritten as

$$s(\mathbf{x}_i, E_i|\boldsymbol{\mu}, \lambda_s, \boldsymbol{\pi}^s) = \sum_{j=1}^{\infty} \pi_j^s \phi(\mathbf{x}_i|\boldsymbol{\mu}_j, \sigma_{E_i}^2) p(E_i|\lambda_s),$$

$$V_j \sim Beta(1, \alpha_s), \qquad \pi_1^s = V_1, \qquad \pi_j^s = V_j \prod_{k=1}^{j-1}(1 - V_k),$$

$$\boldsymbol{\mu}_j \sim \mathcal{U}(\mathcal{X}), \qquad \lambda_s \sim \mathcal{U}(1, 4).$$

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Nonparametric Bayes: why?

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- *Jones et al (2015)* propose to model $s(\mathbf{x}_i, E_i)$ as a finite mixture, and the number of clusters is selected through reversible jump MCMC.
    1. with BNP, $K \to \infty$ as $n \to \infty$: the number of clusters grows with the sample size.
    2. Simulation algorithm for BNP are simple to implement and explore the parameter space faster than the reversible jump MCMC.

Modelling the background

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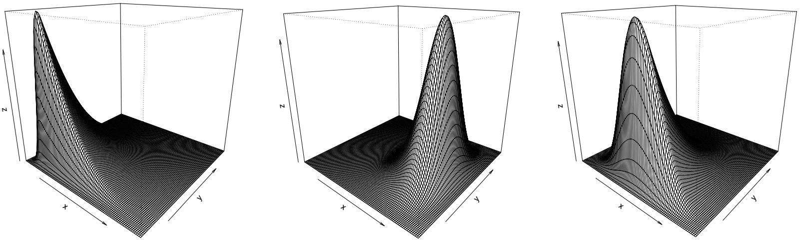
Some characteristics of the background contamination:

- it has an **irregular and unpredictable behaviour**;
- it tends to be **smoother than the sources**;
- **no parametric models are available** to account for it.

The B-spline basis function



- For $x \in \mathbb{R}$, a **B-spline basis function** of order m is defined as

$$B_m(x|\boldsymbol{\xi}) = \frac{x-\xi_1}{\xi_{m+1}-\xi_1} B_{m-1}(x|\boldsymbol{\xi}_{1:m}) + \frac{\xi_{m+1}-x}{\xi_{m+1}-\xi_2} B_{m-1}(x|\boldsymbol{\xi}_{2:(m+1)}),$$

  where $B_1(x|a,b) = I(a \le x \le b)$.
- $B_m(\cdot|\cdot)$ is always positive, unimodal and simple to normalize.

Modelling $b(\cdot)$: a (Bayesian) nonparametric approach

- Given $\boldsymbol{l}_j = (l_{j1}, \ldots, l_{j5})$ and $\boldsymbol{b}_j = (b_{j1}, \ldots, b_{j5})$, we define

$$\mathcal{P}(\mathbf{x}_i | \boldsymbol{l}_j, \boldsymbol{b}_j) \propto B_4(x_i | \boldsymbol{l}_j) B_4(y_i | \boldsymbol{b}_j).$$

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Our proposal

$$b(\mathbf{x}_i, E_i|\mathcal{G}, \lambda_b) = k(\mathbf{x}_i|\mathcal{G})g(E_i|\lambda_b),$$

where

$$k(\mathbf{x}_i|\mathcal{G}) = \int \mathcal{P}(\mathbf{x}_i; \boldsymbol{l}, \boldsymbol{b})\mathcal{G}(d\boldsymbol{l}, d\boldsymbol{b}), \qquad \mathcal{G} \sim \mathcal{DP}(\alpha_b, \mathcal{G}_0).$$

Background model: an alternative representation

- The model can be rewritten as

$$b(\mathbf{x}_i, E_i | \boldsymbol{l}, \boldsymbol{b}, \lambda_b, \boldsymbol{\pi}^b) = \sum_{j=1}^{\infty} \pi_j^b \mathcal{B}_4(x_i | \boldsymbol{l}_j) \mathcal{B}_4(y_i | \boldsymbol{b}_j) g(E_i | \lambda_b),$$

$$U_j \sim Beta(1, \alpha_b), \qquad \pi_1^b = U_1, \qquad \pi_j^b = U_j \prod_{k=1}^{j-1} (1 - U_k),$$

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$$\begin{aligned} l_{j1} &\sim \mathcal{U}(x_{\min}, x_{\max}), \qquad l_{jk} \sim \mathcal{U}(l_{j(k-1)}, x_{\max}), \\ b_{j1} &\sim \mathcal{U}(y_{\min}, y_{\max}), \qquad b_{jk} \sim \mathcal{U}(b_{j(k-1)}, y_{\max}), \quad k = 2, \ldots, 5 \end{aligned}$$

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$$\lambda_b \sim Unif(1, 4).$$

Distinguish between sources and background

Proposition

Let X be a random variable with a density function corresponding to $B_m(\cdot|\boldsymbol{\xi})$. Then

$$Var(X) = \frac{\sum_{p=1}^{m} \sum_{q=p+1}^{m+1} (\xi_p - \xi_q)^2}{(m+1)^2(m+2)}.$$

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$$Var(X) = \frac{\sum_{p=1}^{m}\sum_{q=p+1}^{m+1}(\xi_p - \xi_q)^2}{(m+1)^2(m+2)}.$$

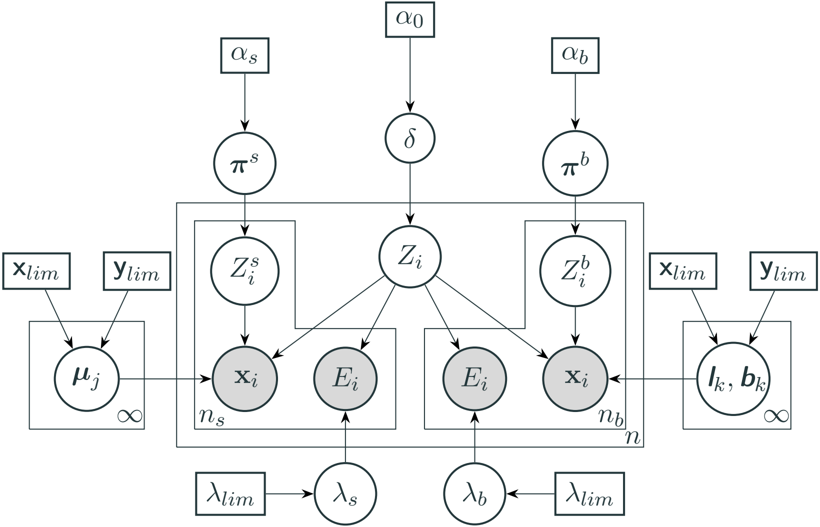
- *We impose a constraint $\forall j$ such that*

$$\frac{\sum_{p=1}^{m}\sum_{q=p+1}^{m+1}(\xi_{jp} - \xi_{jq})^2}{(m+1)^2(m+2)} > \psi,$$

with $\boldsymbol{\xi}_j = \{\boldsymbol{l}_j, \boldsymbol{b}_j\}$ and, given $c > 1$,

$$\psi = c \cdot \max_i \sigma^2_{E_i}.$$

The statistical model: a graphical representation



A suitable MCMC algorithm

- Let

  $\mathcal{S}^{(t)} = \{i = 1, \ldots, n : Z_i^{(t)} = 1\}$ and $\mathcal{B}^{(t)} = \{i = 1, \ldots, n : Z_i^{(t)} = 0\}$.
- We admit at most k_s components for s and k_b components for b.

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- For $t = 1, \ldots, T$, iterate the followings:

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- For $t = 1, \ldots, T$, iterate the followings:
  1. pull photons into $\mathcal{S}^{(t)}$ and $\mathcal{B}^{(t)}$ and update
     $\delta^{(t)} | \cdots \sim Beta(\alpha_0 + |\mathcal{S}^{(t)}|, \alpha_0 + |\mathcal{B}^{(t)}|)$;

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  2. update the weights

$$V_j | \mathcal{S}^{(t)} \sim Beta(n_j^{(t)} + 1, \sum_{k>j} n_k^{(t)} + \alpha_s) \Longrightarrow {\pi^s}_j^{(t)} | \mathcal{S}^{(t)}, \quad j = 1, \ldots, k_s,$$

$$U_j | \mathcal{B}^{(t)} \sim Beta(n_j^{(t)} + 1, \sum_{k>j} n_k^{(t)} + \alpha_b) \Longrightarrow {\pi^b}_j^{(t)} | \mathcal{B}^{(t)}, \quad j = 1, \ldots, k_b.$$

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$$U_j|\mathcal{B}^{(t)} \sim Beta(n_j^{(t)} + 1, \sum_{k>j} n_k^{(t)} + \alpha_b) \Longrightarrow {\pi^b}_j^{(t)} | \mathcal{B}^{(t)}, \quad j = 1, \dots, k_b.$$

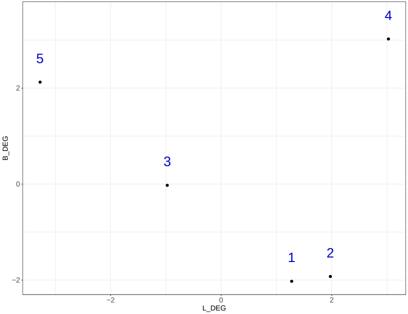
  3. update

$$(\boldsymbol{\mu}_j, \lambda_s)^{(t)} | \mathcal{S}^{(t)}, \dots \qquad \forall j = 1, \dots, k_s,$$

$$(\boldsymbol{l}_j, \boldsymbol{b}_j, \lambda_b)^{(t)} | \mathcal{B}^{(t)}, \dots \qquad \forall j = 1, \dots, k_b.$$

Application on a simulated dataset

- Let $\mathcal{D}$ be a $200 \times 200 \times 25$ array:
  - > $\mathcal{X} = (-4.975, 5.025) \times (-4.975, 5.025)$; each spatial bin is large 0.05.
  - > Energy divided into 25 log10-spaced bins in the range $(1GeV,\ 316.2278GeV)$
- The dataset consists in a background component and 5 sources in the following locations:



Application on a simulated dataset

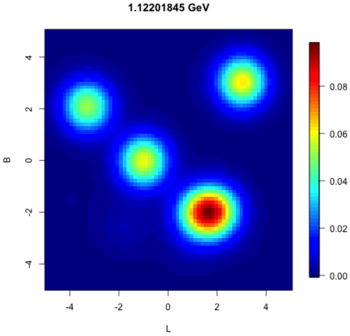
- We sample the counts from each source using a tabulated Point Spread Function and differential flux

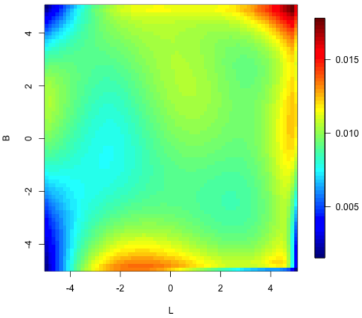
$$\frac{\partial F}{\partial E} = F_0 \left(\frac{E}{E_0} \right)^{-\lambda}, \qquad F_0 = 1 \cdot 10^{-9},\ \lambda = 2,\ E_0 = 1GeV.$$

- A subset of 10000 photons is used.
- Hyperparameters are chosen to be as follows:

$$\alpha = 1, \qquad \alpha_s \sim Gamma(9, 3) \qquad \alpha_b = 1.$$

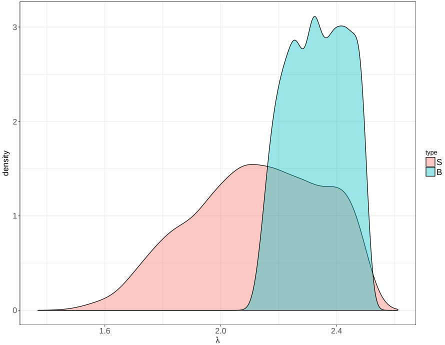
Density estimation





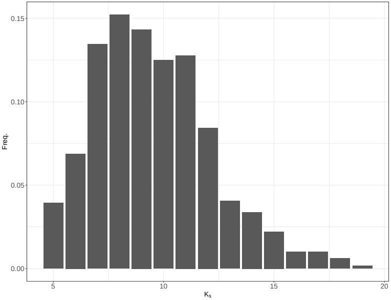
Posterior results: spectral parameters

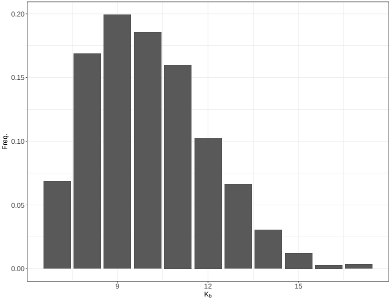
- The spectral parameters λ_s and λ_b are.



Posterior results

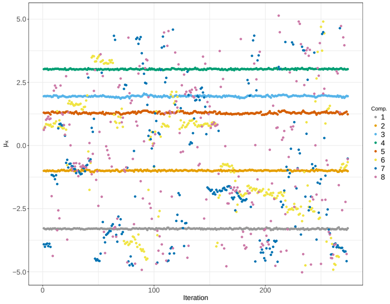
- Number of active components inside the two mixtures after burn-in:

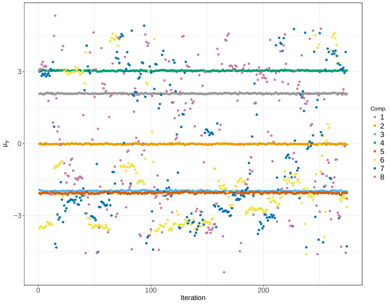




Posterior results

- Given $K_s = \arg\max_k \sum_t I(K_s^{(t)} = k)$, the draws from the posterior distribution of $\boldsymbol{\mu}$ under K_s are:





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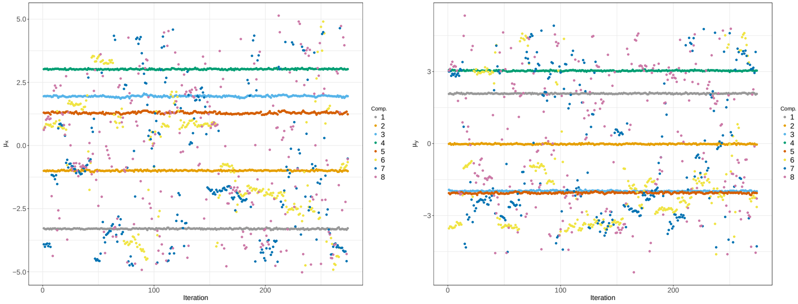
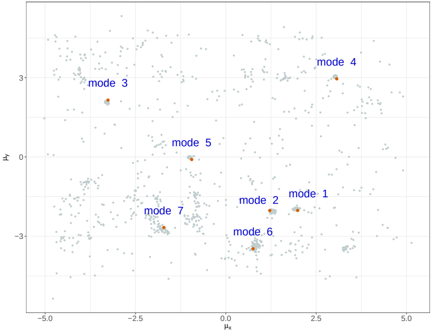


A post-processing algorithm to quantify the information from the posterior distribution of $\boldsymbol{\mu}$ is required.

Post processing algorithm: stage 1

- $\{\boldsymbol{\mu}_1^{(t)}, \ldots, \boldsymbol{\mu}_{K_s}^{(t)}\}$ be the set of draws from the posterior distribution of $\boldsymbol{\mu}$ when the number of active clusters is K_s.



- fit a nonparametric density (or alternatively a 3D-histogram) to determine the most relevant points in the previous map: $(\mathbf{m}_1, \ldots, \mathbf{m}_p)$.

Post processing algorithm: stage 2

- For each $\boldsymbol{\mu}_k^{(t)}$, find the point such that

$$\min_{j=1,\ldots,K_s} ||\boldsymbol{\mu}_k^{(t)} - \mathbf{m}_j|| < r, \tag{1}$$

where r is a given threshold.

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$$\min_{j=1,\ldots,K_s} ||\boldsymbol{\mu}_k^{(t)} - \mathbf{m}_j|| < r, \tag{1}$$

  where r is a given threshold.
- If there exists j which satisfies (1), label $\boldsymbol{\mu}_k^{(t)}$ as j.

Post processing algorithm: stage 2

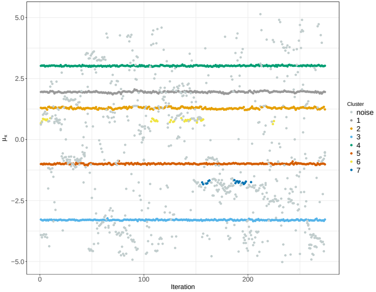
- For each $\boldsymbol{\mu}_k^{(t)}$, find the point such that

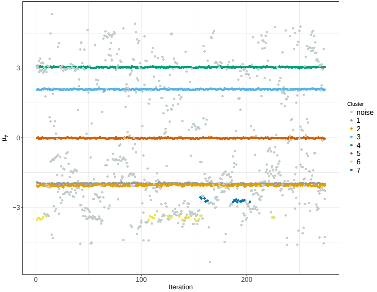
$$\min_{j=1,\ldots,K_s} ||\boldsymbol{\mu}_k^{(t)} - \mathbf{m}_j|| < r, \tag{1}$$

  where r is a given threshold.
- If there exists j which satisfies (1), label $\boldsymbol{\mu}_k^{(t)}$ as j.
- If no j satisfies (1), label $\boldsymbol{\mu}_k^{(t)}$ as *noise*.

Post processing algorithm: stage 2

- The relabelled draws from $\boldsymbol{\mu}$ are:





Post processing algorithm: stage 2

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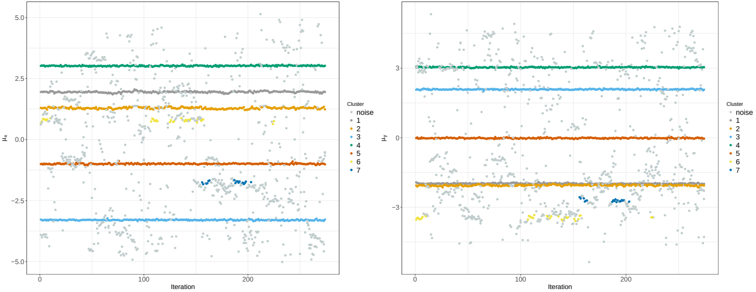


- The new labels can be applied also to $(\pi_1^s, \ldots, \pi_{K_s}^s)^{(t)}$ to have an estimate on the intensity of each cluster.

Post processing algorithm: stage 2

cluster	#counts	$\mathbb{E}(N\|\dots)$	2.5%	25%	75%	97.5%	$\mathbb{P}(source)$
1*	136	215.631	150.475	196.000	238.000	274.000	1.000
2*	166	182.073	123.000	162.250	205.750	237.950	0.956
3*	160	185.511	147.650	173.250	198.000	225.000	1.000
4*	138	231.967	178.825	211.250	252.000	282.350	1.000
5*	149	220.270	173.825	206.000	234.750	263.350	1.000
6	//	19.324	1.000	10.250	29.000	45.350	0.124
7	//	43.056	23.425	34.500	51.500	64.575	0.066

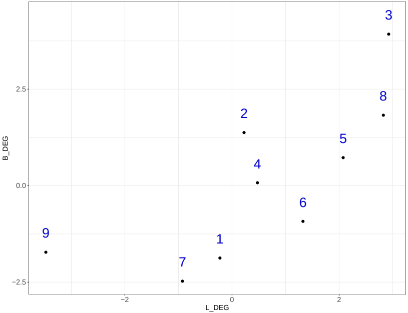
*: the cluster coincides with a real source.

A second simulated dataset

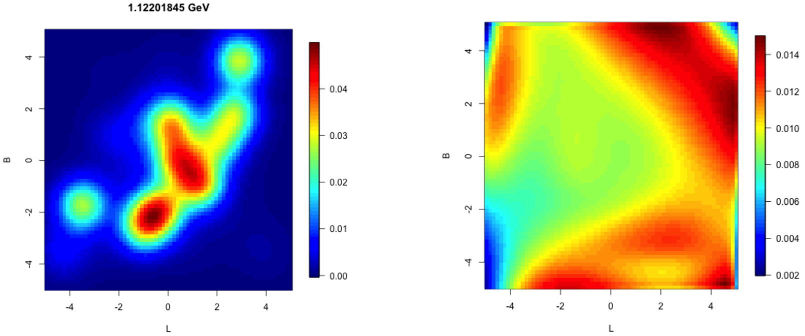
- A subset of 10000 photons is used.
- Hyperparameters are chosen to be as follows:

$$\alpha = 1, \qquad \alpha_s \sim Gamma(9,3) \qquad \alpha_b = 1.$$

- The dataset consists in a background component and 9 sources in the following locations:

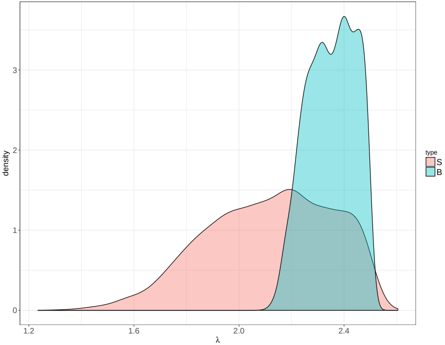


Density estimation



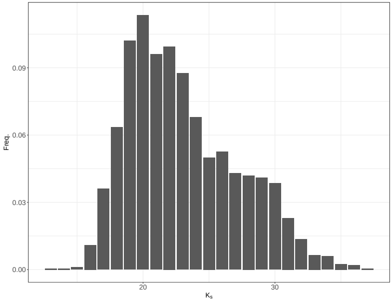
Posterior results: spectral parameters

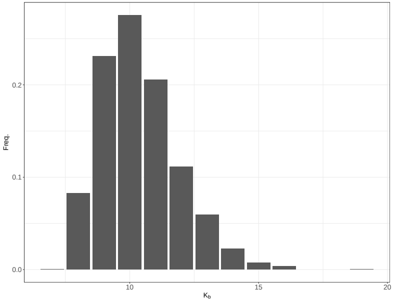
- The spectral parameters λ_s and λ_b are.



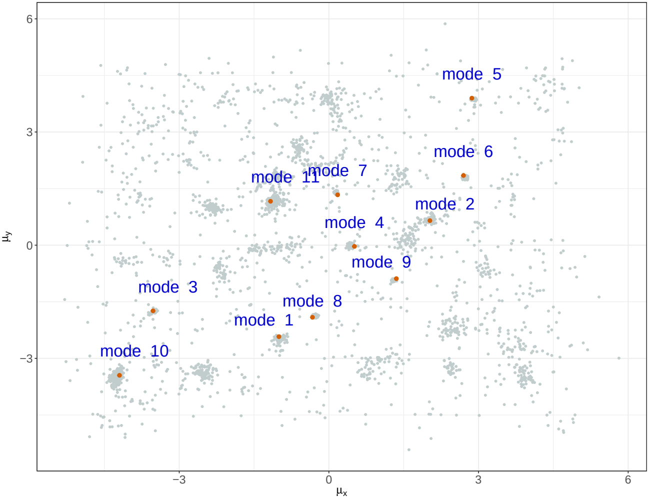
Posterior results: number of components

- Number of active components inside the two mixtures after burn-in:

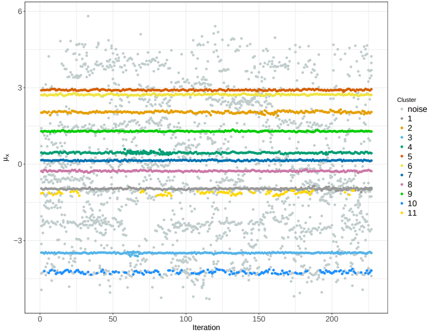


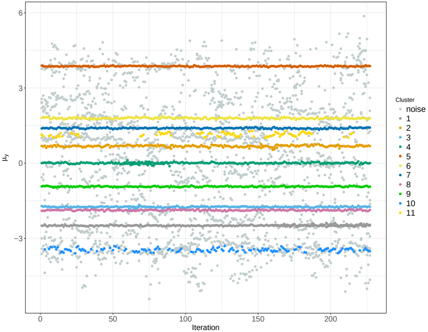


Applying the post processing algorithm, step 1



Applying the post processing algorithm, step 2





Results

cluster	#counts	$\mathbb{E}(N\|\dots)$	2.5%	25%	75%	97.5%	$\mathbb{P}(source)$
1*	138	202.295	142.000	186.000	222.000	252.000	1.000
2*	163	145.190	105.500	134.000	158.000	180.000	0.996
3*	138	174.956	135.650	163.000	186.500	206.700	1.000
4*	148	210.665	156.900	197.000	224.000	253.000	1.000
5*	126	184.093	131.000	167.000	201.500	234.000	1.000
6*	139	165.066	119.650	151.500	179.000	205.000	1.000
7*	141	191.802	140.650	178.500	208.000	241.350	1.000
8*	141	196.612	148.600	184.500	211.000	238.700	1.000
9*	160	214.907	158.300	202.000	230.000	258.700	1.000
10	//	36.884	10.125	31.000	45.000	62.000	0.643
11	//	41.057	14.350	27.750	51.750	91.300	0.388

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Conclusions

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- Our method is capable of:
    1. reconstructing the background component;
    2. locating the possible sources in the map;
    3. quantifying their intensities.
- Even if the posterior distribution of the source model parameters is multimodal, we can quantify the intensity of each mode and estimate its probability of being a source.

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