Joint Spectral-Temporal Analysis of High-Energy Astronomical Sources

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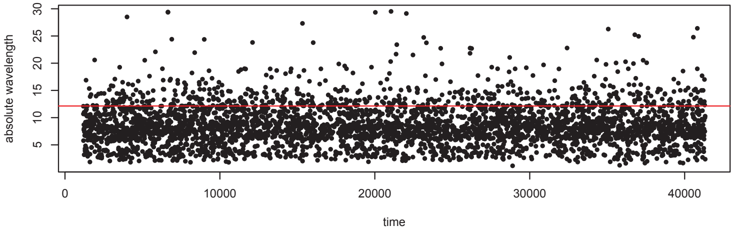
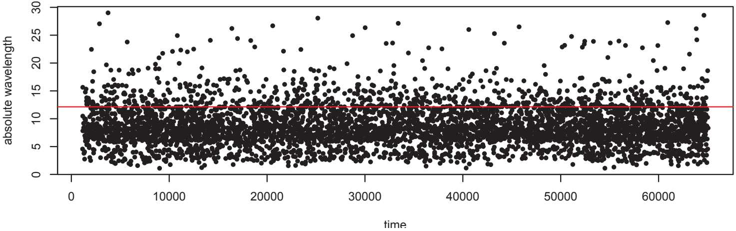
the problem

- we deal with high-spectral/high-temporal resolution grating data
- these data are obtained as lists of photons
- for each photon we know
  1. the time at which it was recorded (t)
  2. its wavelength (w) and hence its energy (E)
- one interesting question: does the distribution of energy change over time?

two typical data sets

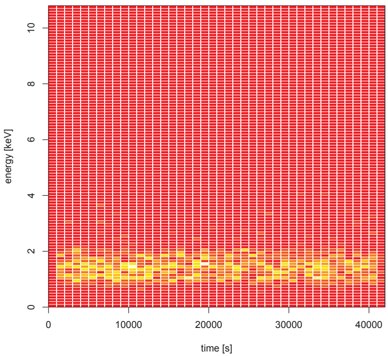


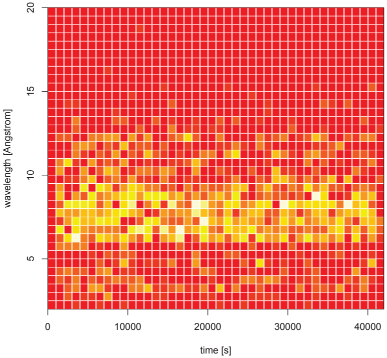


preprocessing: binning the data

- as a first step, we "bin" the data
- i.e., lay a grid over the data and count how many points in each grid box
- Poisson counts in each grid box (or bin)
- size of grid/bin: needs to be carefully chosen

binned data sets





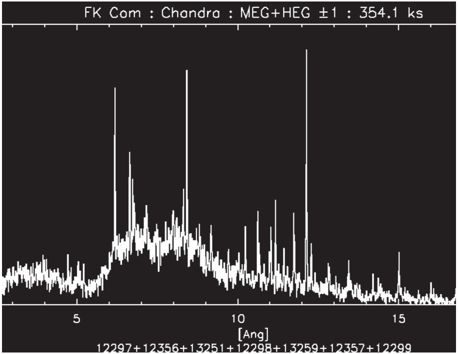
poisson modeling

- each bin is indexed by two quantities:
  1. t: time
  2. w: wavelength
- denote the observed counts as $C(t, w)$
- denote the brightness of a source as $\mu(t, w)$ (expected counts per unit area)
- $$C(t, w) \sim \text{Poisson}\left\{\delta t \times \delta w \times \mu(t, w) \sum_{k=1}^{K} A_k(w)\right\}$$

  K: number of detectors; $A_k(w)$: effective area for the kth detector (all known)

about $\mu(t, w)$

- no simple parametric models, so do nonparametric
- which typically requires smoothness assumption
- with *emission lines*, $\mu(t, w)$ is not completely smooth



modeling of $\mu(t, w)$

- for now assume $\mu(t, w)$ is the same for all t
- i.e., homogeneous across time
- and model the energy spectrum $\mu(w)$
- split $\mu(w)$ into two parts: smooth part + emission lines
    1. smooth part: radial basis expansion
       (use polynomial of power 3: $1, x, x^2, x^3, |x - \text{"knots"}|^3$)
    2. emission lines: delta functions

model for $\mu(w)$

- $$g(\mu(w)) = \sum_{j=1}^{P} \beta_j b_j(w) + \sum_{i=1}^{n} \eta_i I_i(w)$$
- g: link function as in GLM/GAM, for Poisson data
- P: number of basis functions, pre-specified
- b_j: the jth basis (radial basis)
- n: number of bins in the w-direction
- I_i: delta function
- β_j's and η_i's: parameters to be estimated
- note: number of parameters > number of observations

parameter estimation

- $$g(\mu(w)) = \sum_{j=1}^{P} \beta_j b_j(w) + \sum_{i=1}^{n} \eta_i I_i(w)$$
- need to set some β_j's and η_i's to zero
- do L_1 penalty (lasso)
- given tuning parameters γ and ρ, estimate $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$ by minimizing

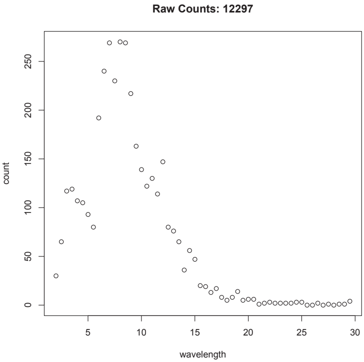
$$-\text{log likelihood} + \gamma\left\{\rho|\boldsymbol{\beta}|_1 + (1-\rho)|\boldsymbol{\eta}|_1\right\}$$

- fast algorithms exist

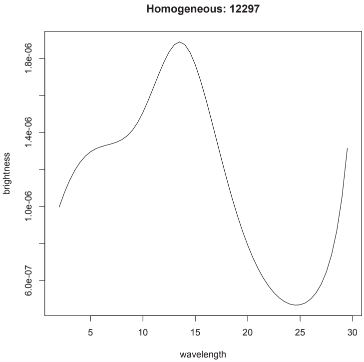
selecting the tuning parameters

- need to choose γ and ρ
- in classical lasso, they can be chosen say by cross-validation, AIC or BIC
- cross-validation: too slow
- AIC/BIC: cannot be blindly used here, as "$p > n$"
- see Chen and Chen (2008, Biometrika), where an Extended BIC criterion is proposed to handle the "$p > n$" issue
- we follow the idea and developed an Extended MDL criterion (Minimum Description Length)

an example (without emission lines)



raw counts



fitted

incorporating time t in modeling

- the energy spectrum typically changes over time
- as a first step, we do piecewise modeling of $\mu(t, w)$
- i.e., $\mu(t, w)$ is the same between any two breakpoints:

$$\begin{aligned}\mu(t,w) = \quad & \mu_1(w) I_{\{t_0 \le t < t_1\}} \\ + \quad & \mu_2(w) I_{\{t_1 \le t < t_2\}} \\ + \quad & \ldots \\ + \quad & \mu_B(w) I_{\{t_{B-1} \le t < t_B\}}\end{aligned}$$

- the number of breakpoints B, and the locations of the breakpoints t_j's, are unknown

selecting B and t_j's

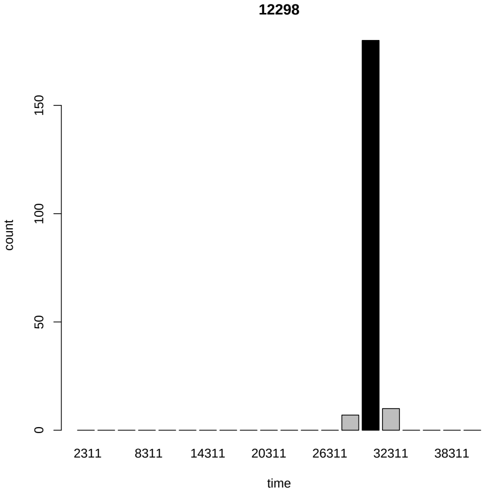
- it is a model selection problem
- MDL has been proven to be very successful in various structural break detection problems
- again, we are in the "$p > n$" scenario
- so direct application of classical MDL won't work here
- as before, we developed an Extended MDL criterion for choosing the final model
- (essentially a penalized likelihood, with 4 penalty terms)

practical fitting

- involves a non-trivial minimization problem
- a possibility is genetic algorithms
- but slow
- we use a “tree growing” strategy
- i.e., at each time step, choose the best location for adding one breakpoint, repeat until a local minimum is found
- (we could certainly do “tree pruning”, and more)

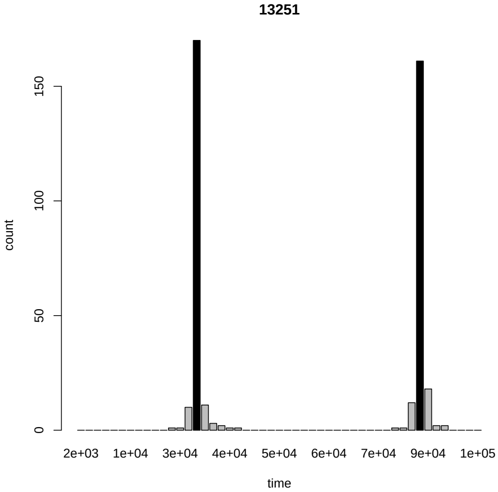
break detection simulation 1

1 true break, with 200 repetitions

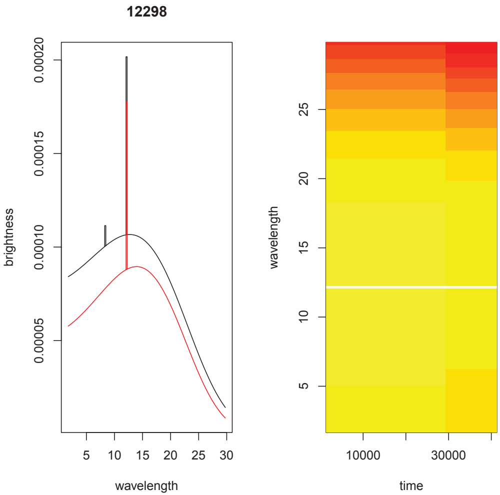


break detection simulation 2

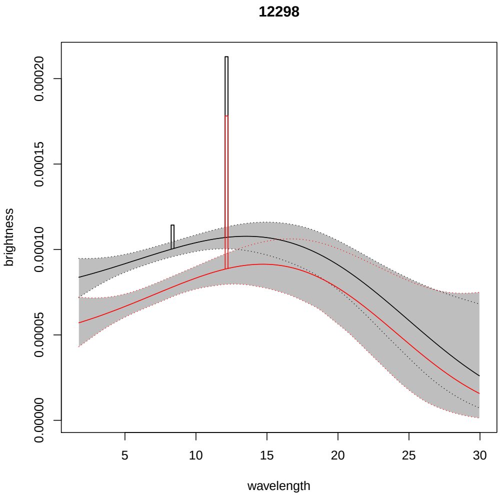
2 true breaks, with 200 repetitions



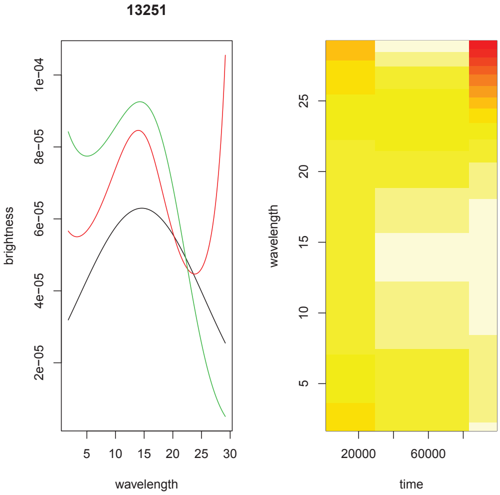
real data set 1



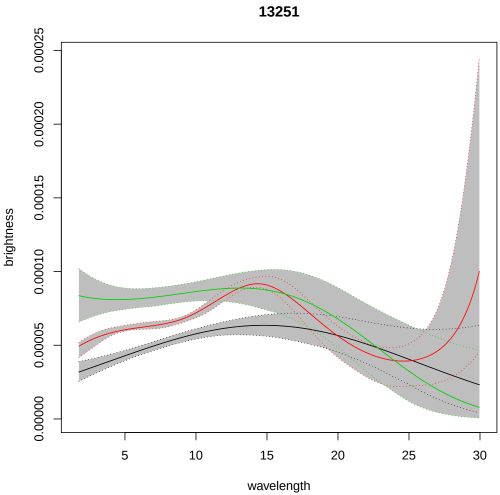
real data set 1



real data set 2



real data set 2



concluding remarks

- presented a method for detecting changes of energy spectrum over time
- modern regression techniques and new model selection criteria are used
- future work:
  1. better modeling in t
  2. theoretical properties of Extended MDL

The end.

Thank you.