

## Fast DEMs for AIA

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Solar Coronal emission at a given temperature is determined by  $n^2(T)$ , the squared density at that temperature integrated along the line of sight. This quantity is termed the Differential Emission Measure (DEM), represented here by  $I(T)$ . Observed intensities,  $C_n$ , in AIA or some other instrument are given by the convolution of an instrument response,  $R_n(T)$ , with the DEM being observed:

$$C_n = \int R_n(T)I(T)dT \quad (1)$$

We present a method for inferring DEMs using data from solar imagers such as AIA. In its basic form, the method is very fast ( $\sim 1$  minute per full disk AIA image), although the DEMs obtained can contain regions of moderately negative emission measure (EM). We demonstrate an extension of the method which removes negative EM while still matching the data. The fidelity of the method is analyzed and it is applied to a coronal loop observed on April 19, 2011.

## Basic Method

Assume that the DEM can be expressed as a linear combination of some set of basis elements,  $B_n$ , so that the integral equation becomes a matrix equation:

$$C_n = \int R_n(T)I(T)dT = \sum_k \left[ i_k \times \int R_n(T)B_k(T)dT \right] \equiv \sum_k i_k \gamma_n A_{nk}, \quad (2)$$

Where  $\gamma_n$  are a set of normalization constants for the response functions, which we choose to be their squared integral, square root. If the number of basis elements is equal to the number of instrument channels, it is straightforward to invert the equation to find the DEM coefficients,  $i_k$ . In that case, the quality of the inversion is highly sensitive to choice of basis.

An alternative is a large number of narrow temperature bins. The underconstrained inversion is resolved by a Singular Value Decomposition (SVD), which picks the coefficient vector with the smallest magnitude (i.e., least squared emission measure).

Remarkably, use of the (square-normalized) instrument response functions as a basis gives identical results. This reduces the number of operations required for each pixel to only  $N^2$  operations with  $N$  instrument channels. It also renders  $A_{ij}$  symmetric.

## Regularization

The basic method's disadvantage is that it can produce negative coefficients and therefore negative EM. Therefore, we enforce a  $\chi^2$  threshold and seek a new DEM which minimizes the sum

$$\chi^2 + \lambda \int [I(T)]^2 dT = \sum_i \frac{\Delta C_i}{\sigma_i^2} + \lambda \sum_{ij} i_i i_j A_{ij}, \quad (3)$$

where  $\chi^2$  has the usual definition,  $I(T)$  is the DEM described above, and  $\lambda$  is a regularization parameter chosen to enforce the desired  $\chi^2$  threshold,  $\chi_0^2$ .

Using our basic DEM solution, we replace  $i_i$  with a set of corresponding regularization corrections to the data values,  $\Delta C_i$  and, so that  $i_i = \sum_j A_{ij}^{-1} (C_i + \Delta C_i) / \gamma_i$ :

$$\chi^2 + \lambda \int [I(T)]^2 dT = \sum_i \frac{\Delta C_i^2}{\sigma_i^2} + \lambda \sum_{ij} (C_i + \Delta C_i) \frac{A_{ij}^{-1}}{\gamma_i \gamma_j} (C_j + \Delta C_j). \quad (4)$$

Assuming  $\Delta C_i \ll C_i$ , the data corrections are given by

$$\lambda^2 = \chi_0^2 / \sum_j \sigma_j^2 \left[ \sum_i \frac{C_i A_{ij}^{-1}}{\gamma_i \gamma_j} \right]^2, \quad \Delta C_j = \lambda \sigma_j^2 \sum_i \frac{C_i A_{ij}^{-1}}{\gamma_i \gamma_j}. \quad (5)$$

## Iteration

This regularization step is very fast, but since the DEM has only as many basis elements as there are channels, it remains liable to producing negative emission in cases with sharp features. We therefore remove the remaining negative emission via the following iterative process:

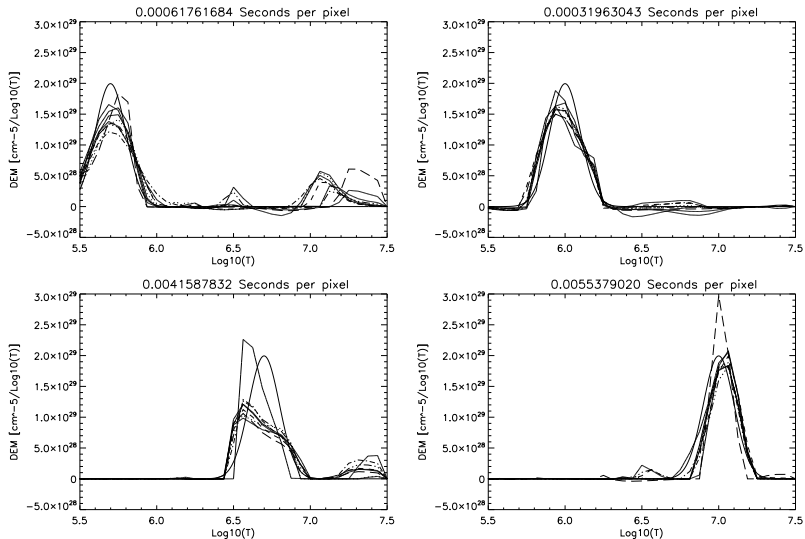
1. Zero the negative EM in the current DEM,  $I^{(n)}$ , to create a new DEM,  $I_+^{(n)}$
2. Compute the data intensities,  $C_i^+$  corresponding to  $I_+^{(n)}$ .
3. Take the difference between  $C_i^+$  and the original  $C_i$ ,  $\Delta C_i^+ = C_i^+ - C_i$ .
4. Compute correction DEM coefficients,  $\Delta i_i = \sum_j A_{ij}^{-1} \Delta C_j^+ / \gamma_j$ .
5. Subtract the corresponding DEM corrections,  $\Delta I^{(n)}$  from  $I_+^{(n)}$ . By construction, this restores  $I^{(n+1)} \equiv I_+^{(n)} - \Delta I^{(n)}$  to agreement with the data, but reintroduces some negative emission.
6. Repeat until  $C_i^+$  matches  $C_i$  to within the desired  $\chi^2$ .

To speed convergence of the iteration, we also employ a linear extrapolation on  $I^{(n)}$ . The extrapolative steps are only taken when they improve  $\chi^2$ . If at first they fail, they are retried with a step size chosen so that positive components of the current DEM are reduced by no more than 150%.

## Performance

- ▶ Detailed in plots on next pages
- ▶ Matches synthetic data well with no negative EM
- ▶ Typical times for test distributions are  $\sim 10^{-3}$  seconds or less per pixel
- ▶ Narrow, high temperature Distributions are the most difficult test cases
- ▶ Average solar DEM times are  $\sim 0.3 \times 10^{-3}$  seconds per pixel
- ▶ No apparent issues obtaining good  $\chi^2$  with solar observations

## Example Test DEM Inversions



**Figure:** Test DEM inversions using Log-normal simulated DEMs, convolved with the AIA response functions, with noise (Solid Lines), each with a width of 0.1.

# $\chi^2$ for Example Inversions

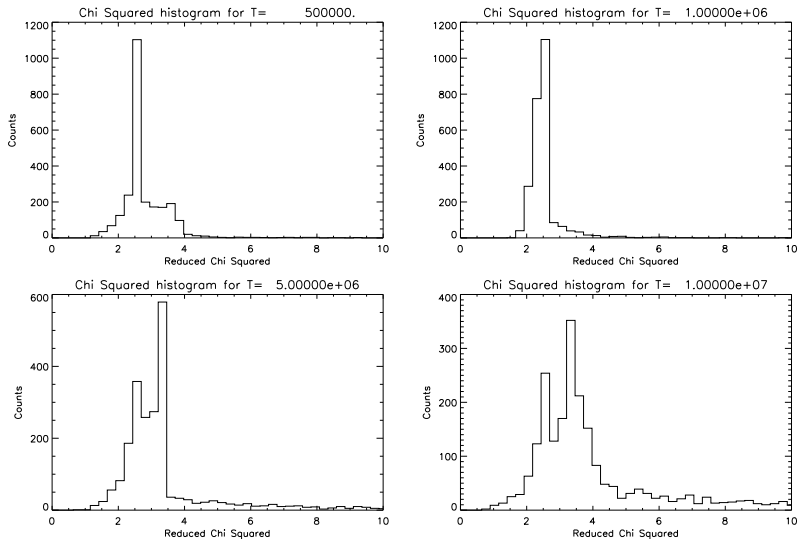
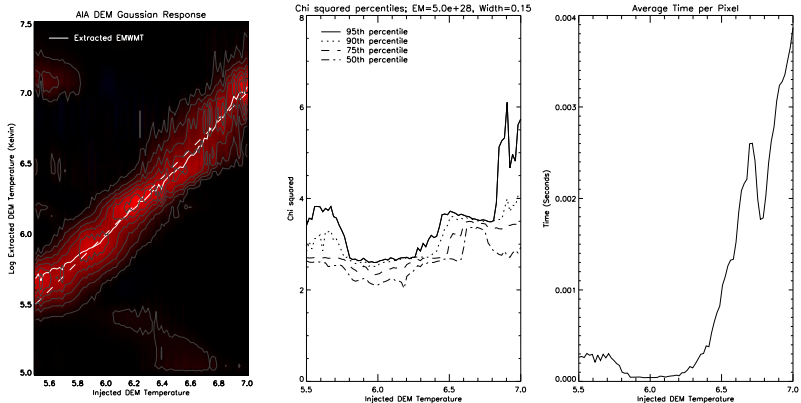


Figure: Residual  $\chi^2$  For 2500 samples corresponding to each of the plots shown on the previous page.

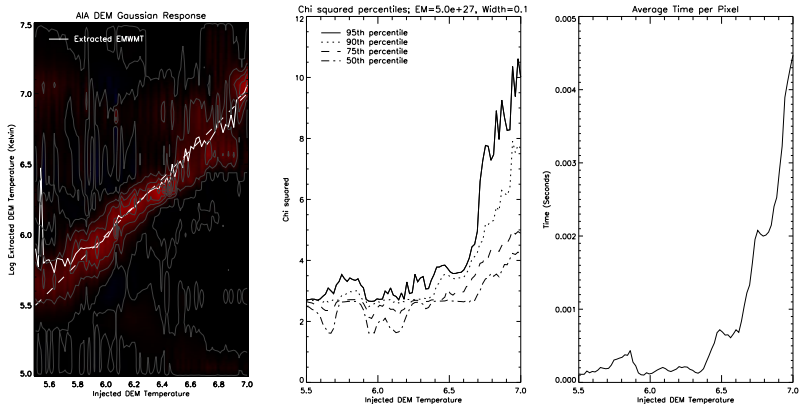
# Broader Distribution, Moderate EM



**Figure:** Response to Log-normal distributions of width 0.15 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted median temperature (EMWMT).

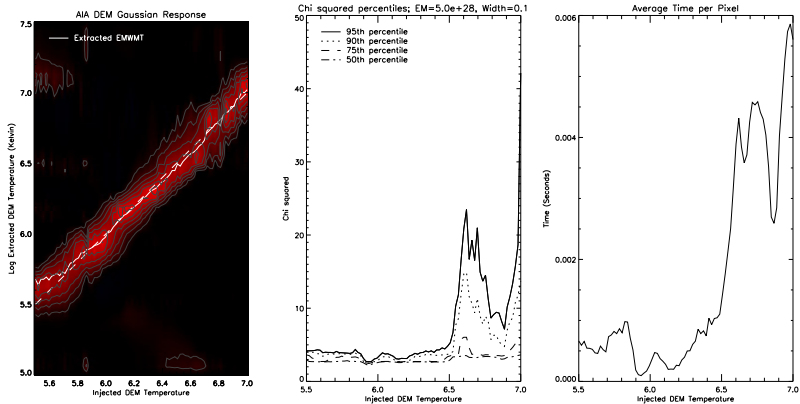


# Narrow Distribution, Lower EM



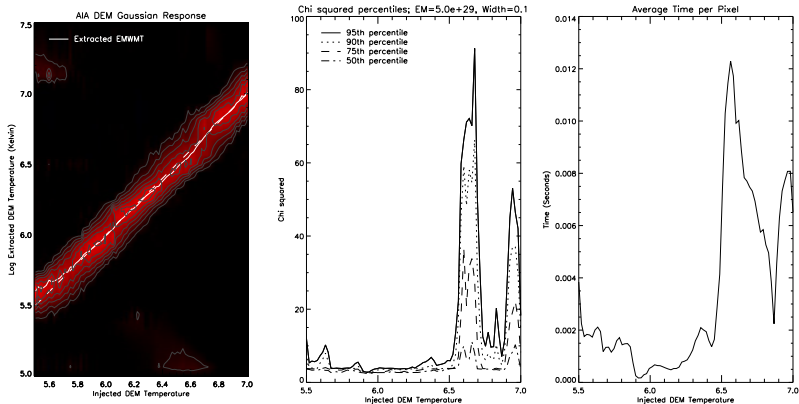
**Figure:** Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{27}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted median temperature (EMWMT).

# Narrow Distribution, Moderate EM



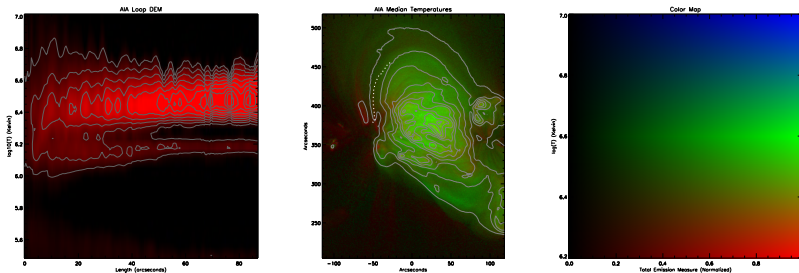
**Figure:** Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{28}$  at temperatures from 5.5 to 7.0 dex. Solid lines on left shows emission measure weighted median temperature (EMWMT).

# Narrow Distribution, Higher EM



**Figure:** Response to Log-normal distributions of width 0.1 and total EM  $5.0 \times 10^{29}$  at temperatures from 5.5 to 7.0 dex. Solid line on left shows emission measure weighted median temperature (EMWT).

# Active Region and Coronal Loop



**Figure:** Middle: EMWMT (hue) and Total EM (intensity) from AIA DEM inversion of Active region (covered by EIS fov on April 19, 2011). Left: detailed DEMs for loop segment indicated by dotted lines on middle plot. No background subtraction has been performed; which part is the loop? Right: color scale for middle plot.

EMWMT Movie of a flare from January 28, 2011 (color scheme shown previous page)