DEM for the statistically challenged
χ^2 *vs.* L1 norm minimization?

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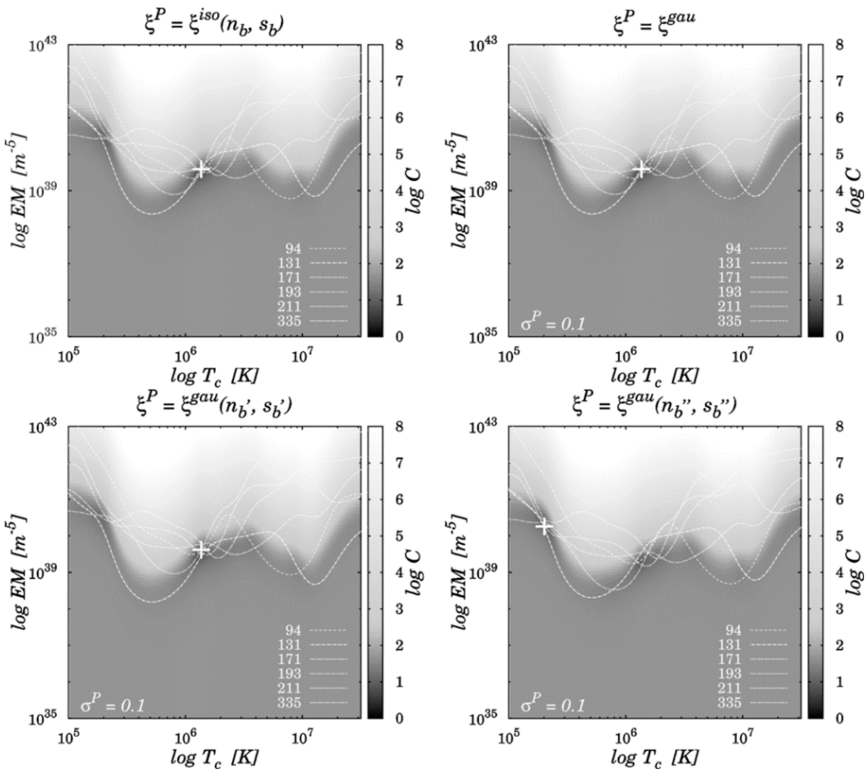
Merit function (a.k.a. objective function, criterion, etc.)

- Many DEM inversion algorithms based on χ^2 minimization $\chi^2 = \min\left[\sum_{b=1}^{N_b}\left(\frac{I_b^{obs} - I_b^{th}(\xi)}{\sigma_{unc}}\right)^2\right]$
- Example of χ^2 merit function for isothermal inversion (DEM $\xi = EM\,\delta(T_c)$)



- Goal is not to derive a minimization algorithm but to understand the properties of the merit function
- Results apply to all χ^2–based inversion schemes
- Fundamental equivalence between noise and multithermality

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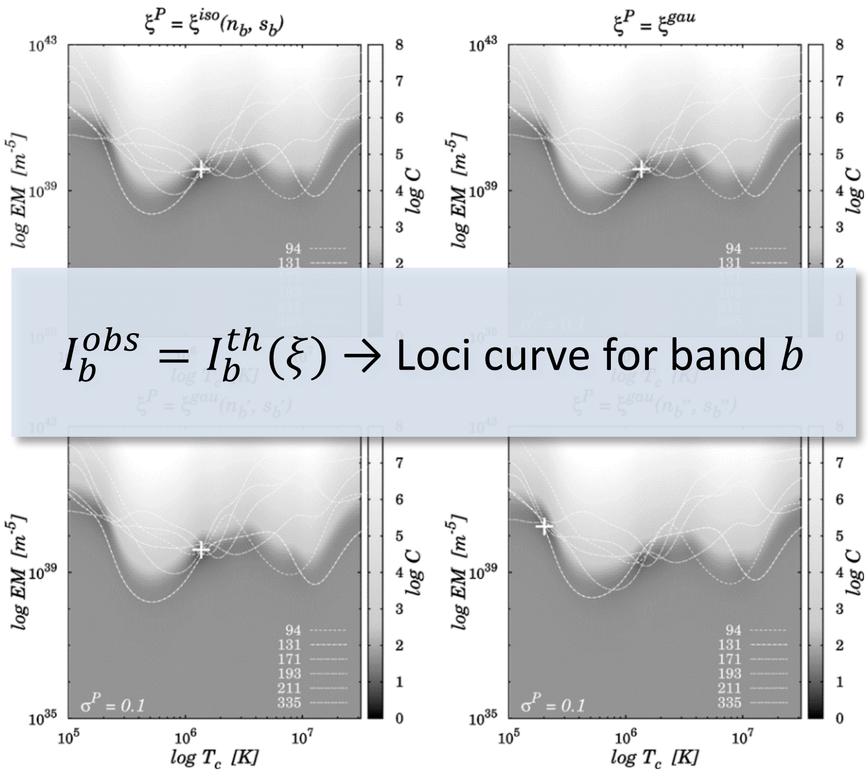

$I_b^{obs} = I_b^{th}(\xi) \rightarrow$ Loci curve for band b

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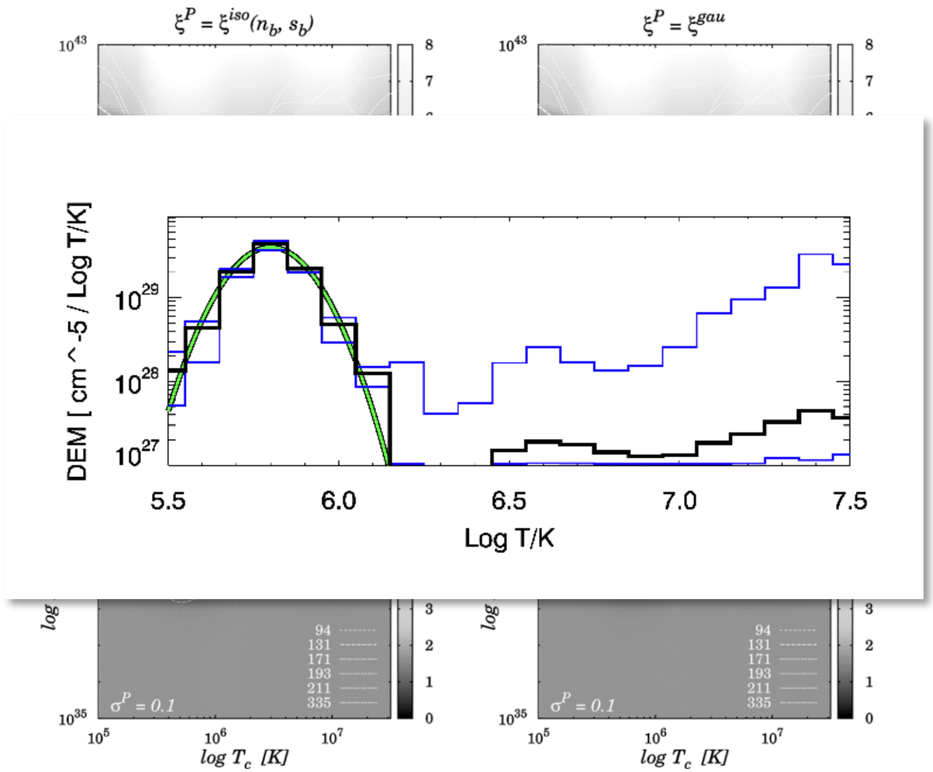

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Chloé's approach

- 6 AIA bands → can't fit a very complex DEM
- Systematic search of all solutions for a simple test case
  - Gaussian (log-normal) DEM plasma input

$$\xi^P_{gau} = \frac{EM}{\sigma\sqrt{2\pi}} \exp\left(-\frac{[\log T_e - \log T_c]^2}{2\sigma^2}\right) = EM \times \mathcal{N}(\log T_e - \log T_c)$$

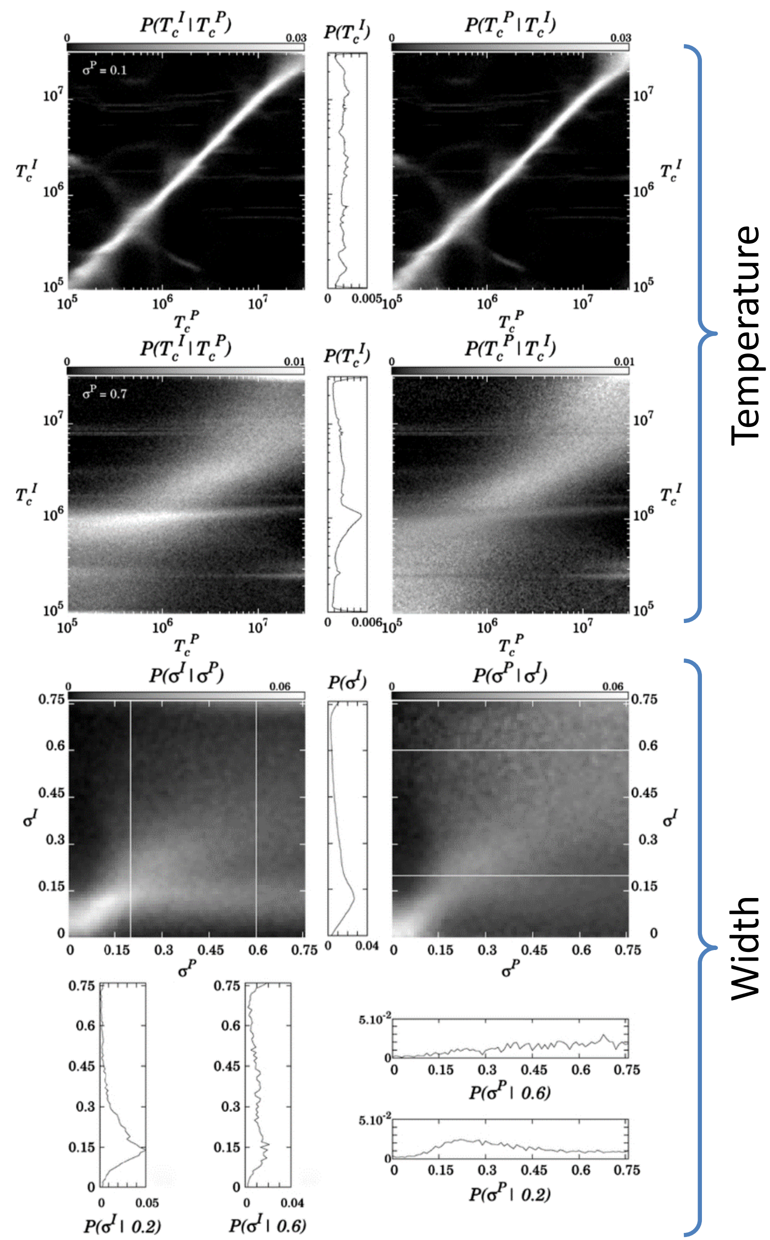
  - Search for Gaussian solutions

$$\chi^2 = \min\left[\sum_{b=1}^{N_b}\left(\frac{I_b^{obs} - I_b^{th}(\xi)}{\sigma_{unc}}\right)^2\right]$$

  - Detailed uncertainties
    - Photon noise
    - 25% calibration & atomic physics

- (some of the) results
  - Broad DEMs poorly constrained
  - Possible bias of the solutions towards
    - $T_c = 1$ MK & $\sigma = 0.1 \log T$
    - Similar to Weber et al. 2005, ApJ, 635, L101

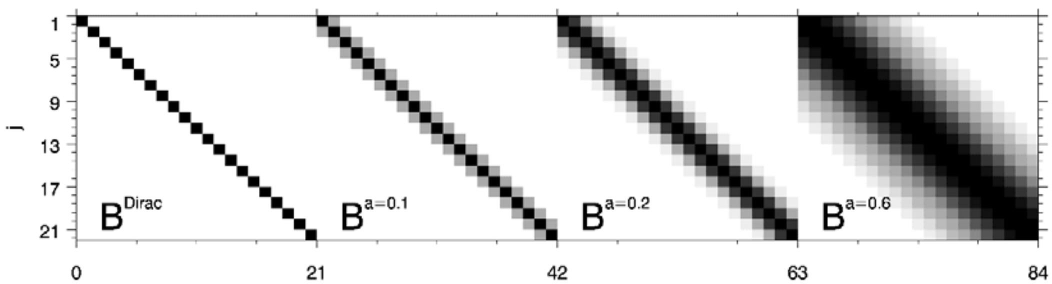
(Guennou, C. et al. 2012a, ApJS, 203, 25,
Guennou, C. et al. 2012b, ApJS, 203, 26)



Alternative to χ^2

- Cheung, M., Boerner, P., Schrijver, C. et al. 2015, *"Thermal Diagnostics with the Atmospheric Imaging Assembly onboard the Solar Dynamics Observatory: A Validated Method for Differential Emission Measure Inversions"*, ApJ, in press

  - Dictionary-based inversion



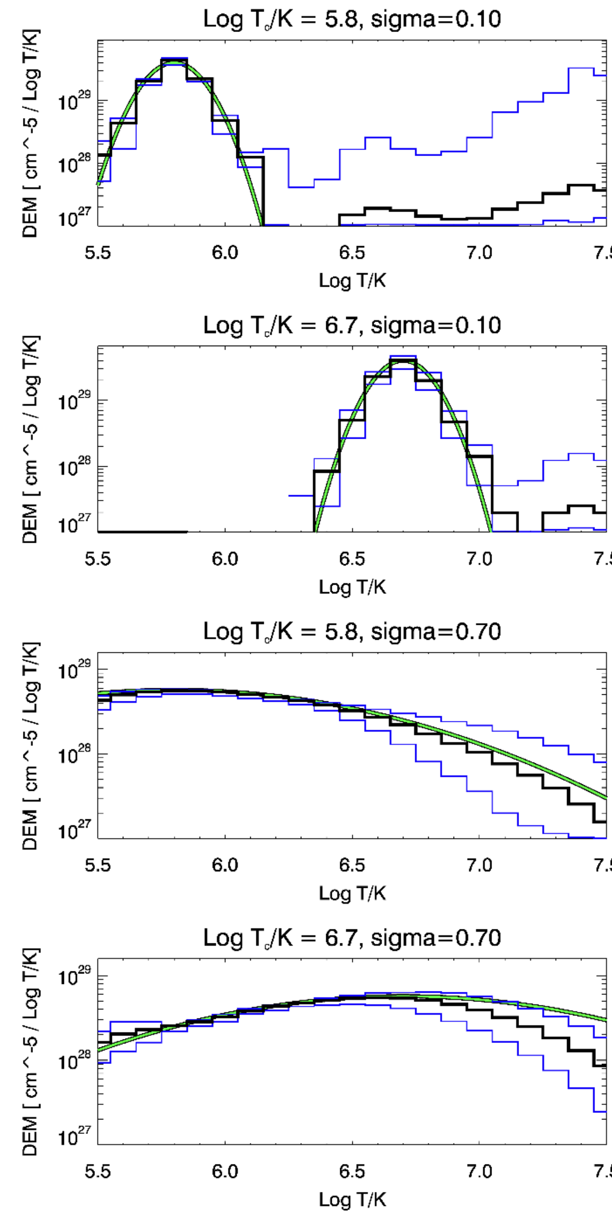
  - Not χ^2-based
  - Minimizes the L1 norm of the coefficients x_j, i.e.

$$\mathrm{LP1:minimize} \sum_{j=1}^{n} x_j \ \text{ subject to}$$

$$\begin{aligned} \mathbf{D}\vec{x} &\leq \vec{y}+\vec{\eta}, \\ \mathbf{D}\vec{x} &\geq \max(\vec{y}-\vec{\eta},0), \\ \vec{x} &\geq 0. \end{aligned}$$

  - If B = B^{dirac} only , $\sum_{j=1}^{n} x_j = EM$

  - More robust than χ^2 for wide DEMs ?



Back to χ^2: why are broad DEMs poorly constrained?

$$\chi^2 = \min\left[\sum_{b=1}^{N_b}\left(\frac{I_b^{obs} - I_b^{th}(\xi)}{\sigma_{unc}}\right)^2\right]$$

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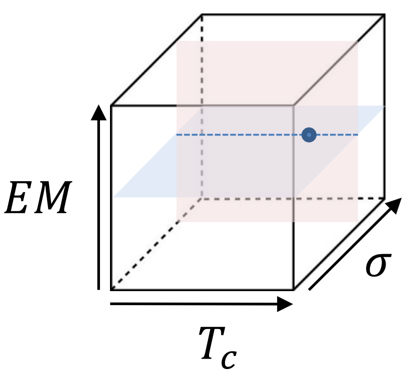

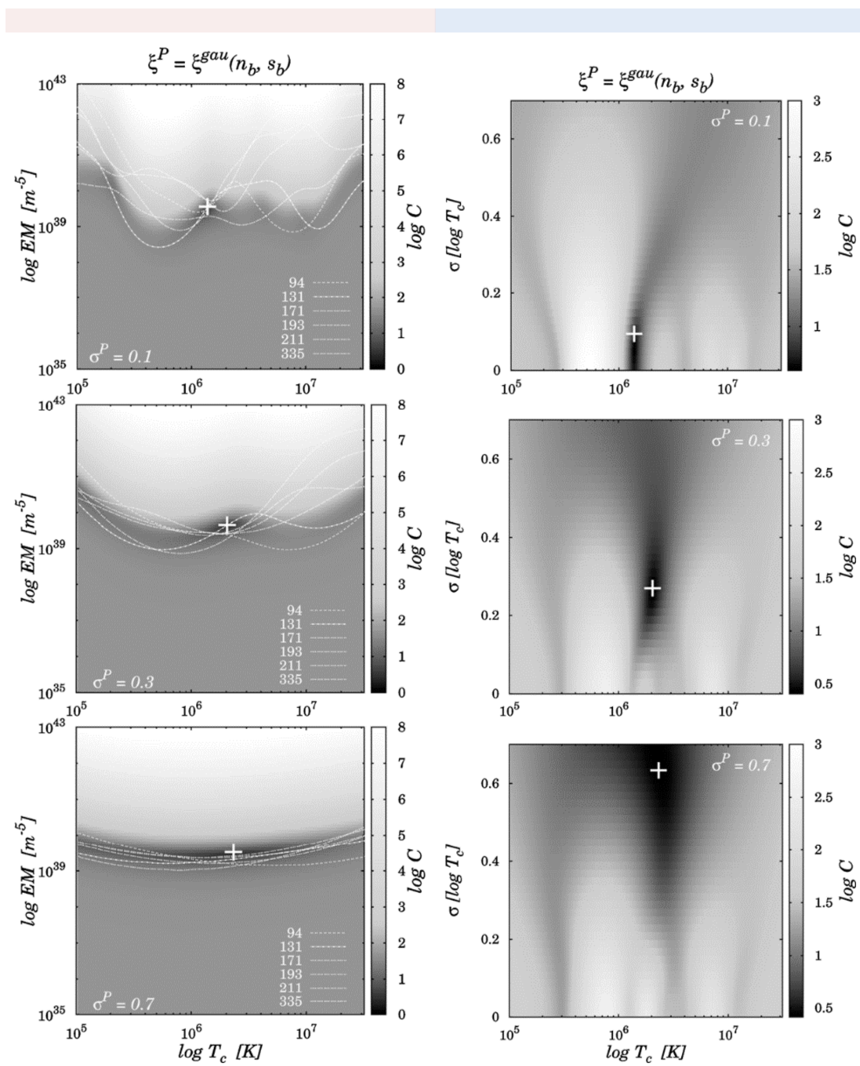

- For ∞ broad DEM

$$I_b^{th} = EM \int_0^{\infty} R_b(T_e)\, \mathrm{d}\log T_e$$

Independent from T_c

(Weber et al. 2005)

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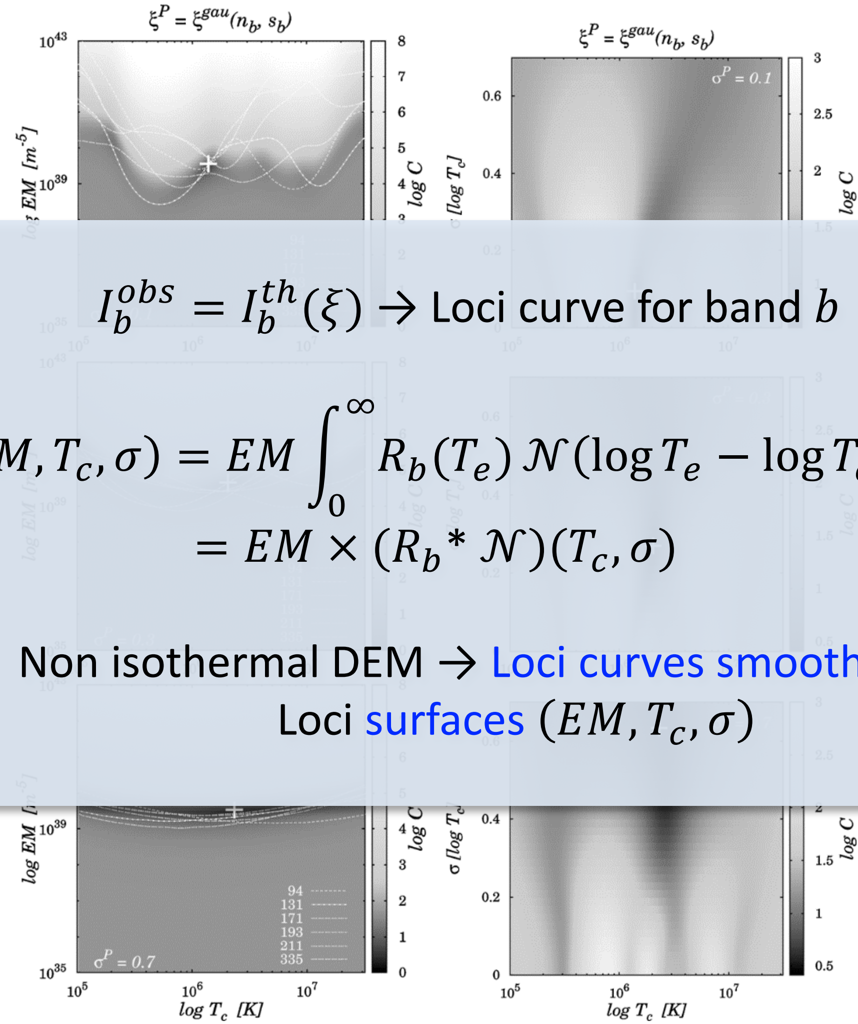

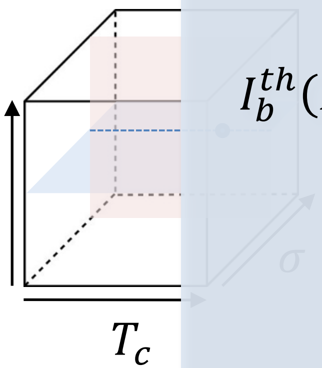

$I_b^{obs} = I_b^{th}(\xi) \rightarrow$ Loci curve for band b

$$I_b^{th}(EM, T_c, \sigma) = EM \int_0^{\infty} R_b(T_e)\, \mathcal{N}(\log T_e - \log T_c)\, \mathrm{d}\log T_e$$
$$= EM \times (R_b * \mathcal{N})(T_c, \sigma)$$

Non isothermal DEM → Loci curves smoothed in T_c

Loci surfaces (EM, T_c, σ)

- For ∞ broad DEM

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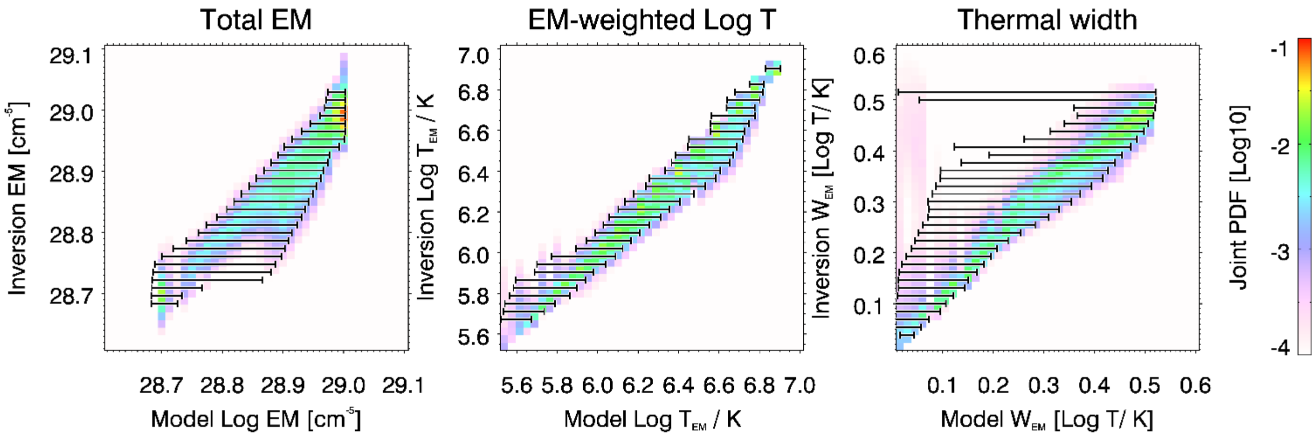
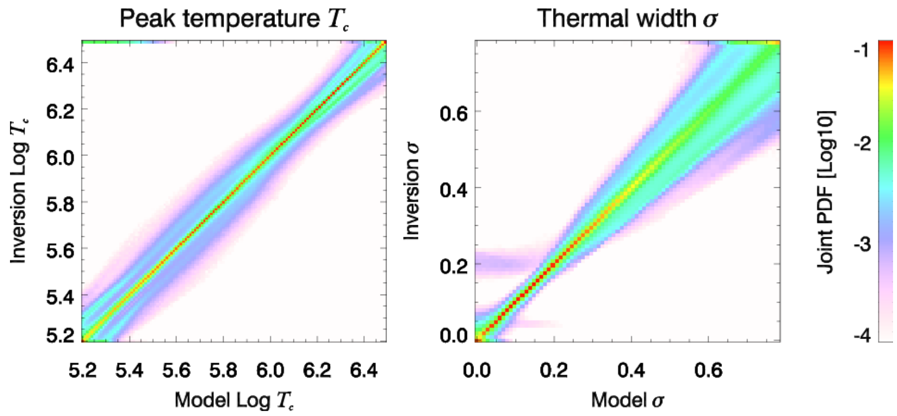
Comparison χ^2 - L1

- Cheung et al. use **`aia_bp_estimate_error`** to estimate the uncertainties
  - photon noise, compression and quantization round-off, error in dark subtraction
  - no atomic physics & calibration uncertainties



- Run of Chloe's code with the same (<<25%) uncertainties (constant $EM = 10^{38}$ m^{-5})
  - Similar results !
  - Need to run Mark Cheung's code with 25% uncertainties



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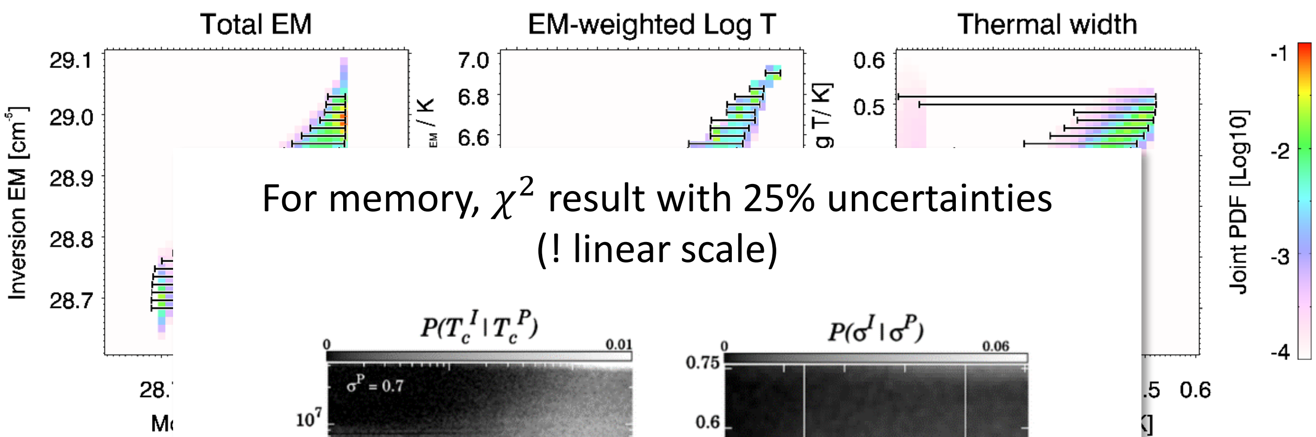
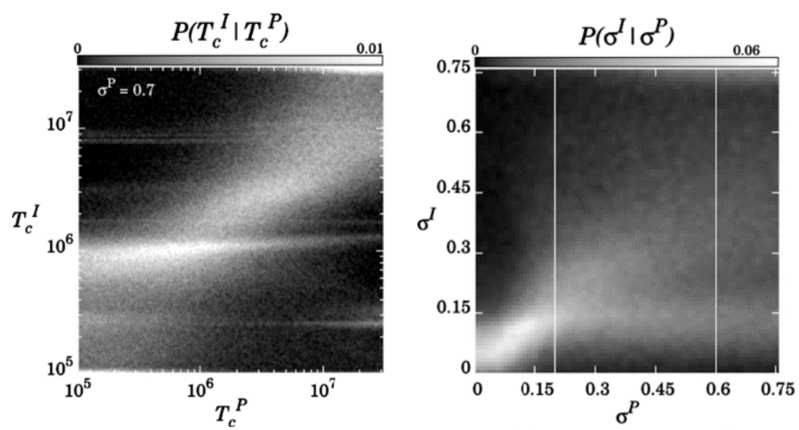
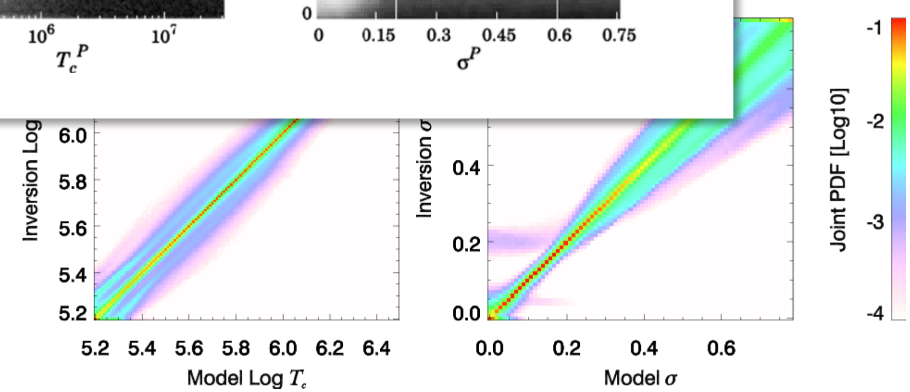


For memory, χ^2 result with 25% uncertainties
(! linear scale)



- Run of Chloe's cod … 10^{38} m^{-5})
  - Similar result
  - Need to run



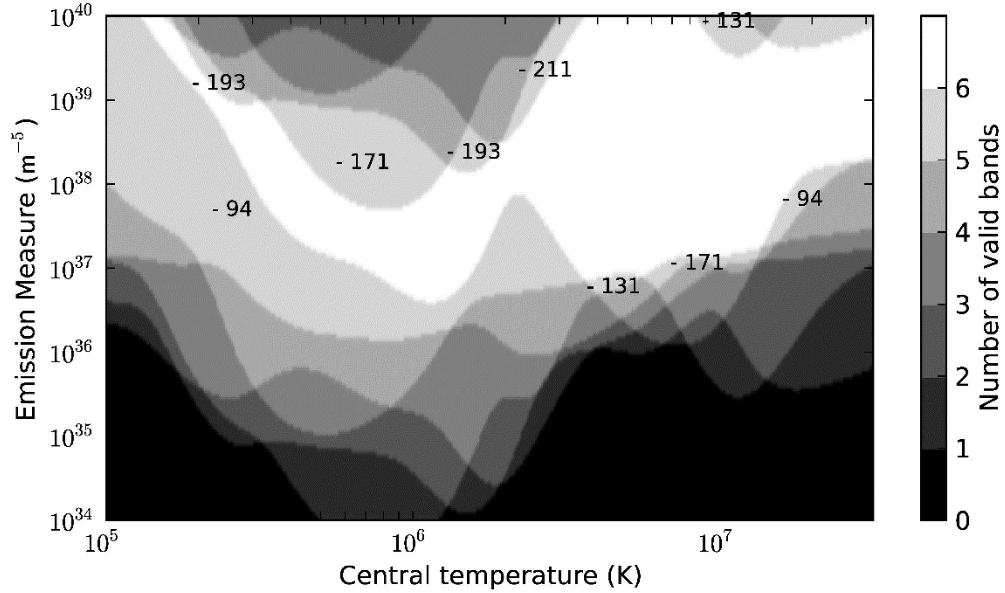
To be continued…

- Mark Cheung's method is all new to me
  - I don't understand yet how the L1 approach can alleviate the difficulties found for broad DEMs
  - That does not mean it's not the case :D

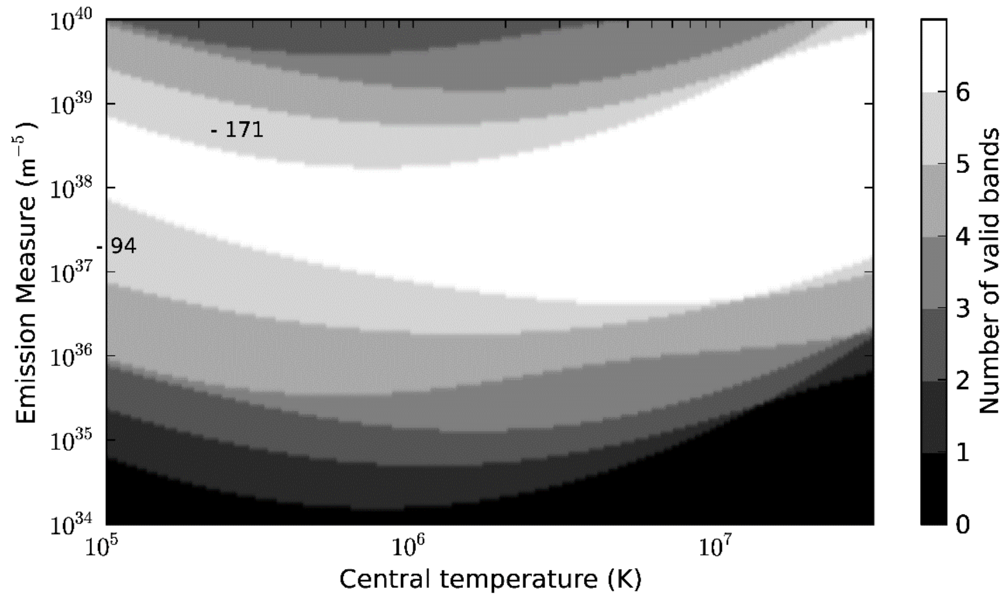
- Discussion started with Mark Cheung
  - Run both codes with the same input DEMs & uncertainties
  - Compare

- ISSI 2016 …

AIA signal vs Temperature & EM



Isothermal



$\sigma = 0.5 \operatorname{Log} T_e$

Is my plasma isothermal?