Fitting Computer Models

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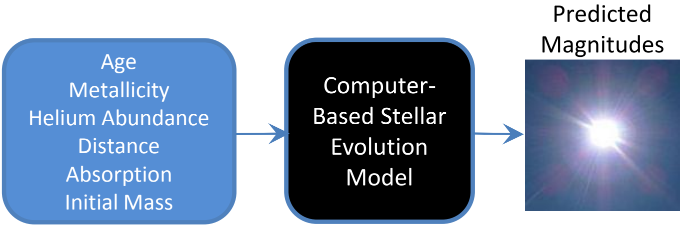
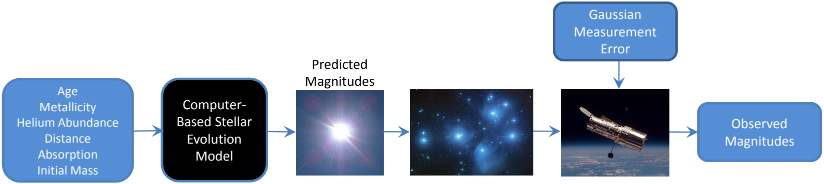
Computer Model for Stellar Evolution



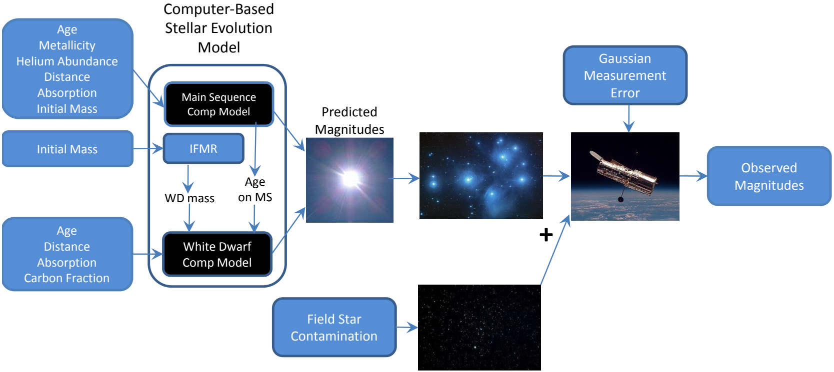
- We observe a star's *photometric magnitudes*—the apparent brightness of a star in several wide wavelength bands.
  - Magnitudes observed with Gaussian measurement error.
- Computer models to predict the photometric magnitudes of a star given a set of input parameters that describe certain characteristics about the star.
- Embed these models in a multilevel model for statistical inference.

Combining Computer Models and Statistical Models



- Observe photometric magnitudes through n different filters per star.
- Model photometric magnitudes as n independent Gaussians.
  - Means involve the computer models for stellar evolution; depend on the stellar evolution parameters.
  - Known Gaussian measurement errors in the covariance matrix.
- Data is contaminated by non-cluster *field stars*.
  - Use a finite mixture model, with field star magnitudes assumed uniform over the range of the data.

Final Combined Computer/Statistical Model



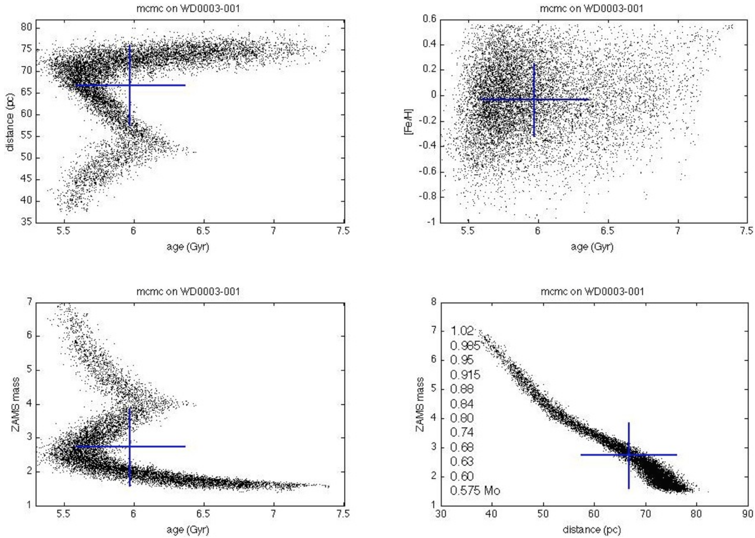
- We take a Bayesian approach to model fitting.
  - Informative prior distributions are constructed based on previous studies and astrophysical theory.

Bayesian Statistical Analyses

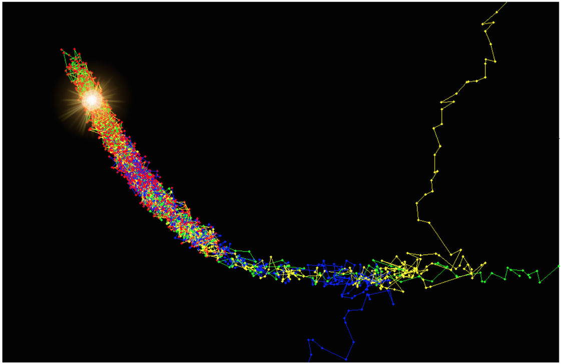
- Specifying a Bayesian statistical model:
  - Likelihood Function: the distribution of the data, Y, given model parameters, Θ. Denoted by $L(\Theta) = P(Y \mid \Theta)$.
    - Θ may contain computer model inputs.
  - Prior Distribution: represents knowledge about the parameters obtained *prior* to the current data. Denoted by $P(\Theta)$.
  - Posterior Distribution: represents knowledge about the parameters in light of the data. Denoted by $P(\Theta \mid Y)$.
- From Bayes' Theorem:

$$P(\Theta \mid Y) \propto P(Y \mid \Theta) P(\Theta)$$

Complex Posterior Distributions



Exploring $P(\Theta \mid Y)$



source: http://commons.wikimedia.org/wiki/File:3dRosenbrock.png#mediaviewer/File:3dRosenbrock.png

- We explore the posterior distribution, $P(\Theta \mid Y)$, using Markov chain Monte Carlo (MCMC) methods.
- MCMC produces (correlated) samples from $P(\Theta \mid Y)$.
- Fitted values, 95% CIs, etc. computed using MCMC draws.

The Metropolis Algorithm

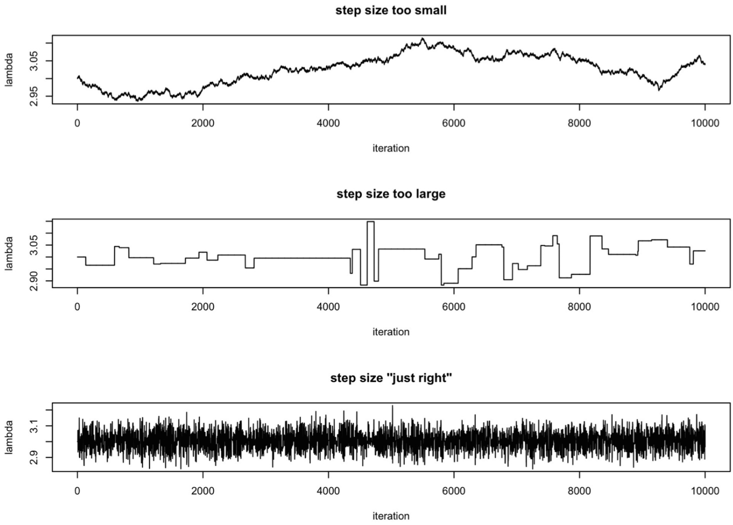
Draw $\Theta^{(0)}$ from some starting distribution.

For $t = 1, 2, \ldots$

- Draw "proposed state" $\Theta^{(*)} = \Theta^{(t-1)} +$ *random perturbation*.
  - random perturbation must be symmetric
  - e.g. $\Theta^{(*)} \sim N\left(\Theta^{(t-1)}, \xi\right)$
- Compute $a = \min\left(1, \frac{P\left(\Theta^{(*)} \mid Y\right)}{P\left(\Theta^{(t-1)} \mid Y\right)}\right)$.
- Set $\Theta^{(t)} = \Theta^{(*)}$ with probability a, else set $\Theta^{(t)} = \Theta^{(t-1)}$.

Note that proposed states "uphill" are always accepted, while proposed states "downhill" are only sometimes accepted.

The Metropolis Algorithm: Step-Size Effect

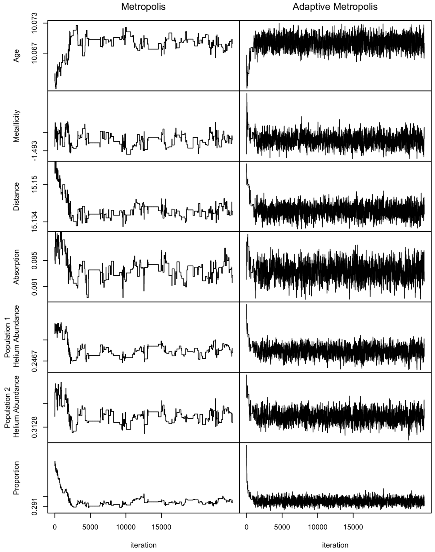


Adaptive Metropolis Algorithm

- How to choose an "optimal" proposal distribution?
- For a $N(0, \Sigma)$ target distribution, the optimal proposal distribution is $N\left(0, \left[(2.38)^2/d\right]\Sigma\right)$, where Σ is a $d-$dimensional covariance matrix (Gelman *et al.* 1996).
- *Adaptive Metropolis* (AM) algorithm (e.g., Haario *et al.* 2001):
  - At iteration t, draw $\Theta^{(*)} \sim N\left(\Theta^{(t-1)}, \left[(2.38)^2/d\right]\xi_{t-1}\right)$.
  - ξ_{t-1} is the empirical covariance matrix of $\Theta^{(0)}, \ldots, \Theta^{(t-1)}$.

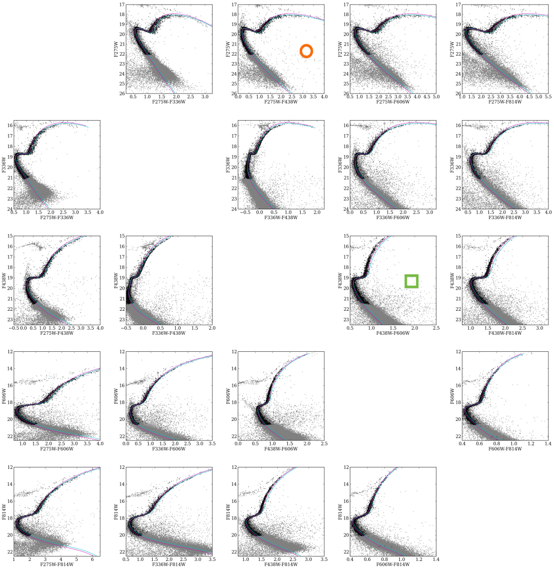
Key condition: the amount of adaptation at iteration t goes to 0 as $t \to \infty$ (Diminishing Adaptation Condition).

Adaptive Metropolis Advantage



- Exploring a (marginal) posterior distribution using an AM algorithm.
- Improved efficiency and convergence compared to non-adaptive Metropolis implementation.
    - Same data and setup used for both algorithms.
    - AM algorithm adapts the proposal distribution starting at iteration 1000.

CMD Matrix with Fitted Computer Models



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