Table 1: Posterior Probabilities for the Grid of the Power-Law Model. The 95% posterior region is indicated in bold face.

		N_H					
		0.250–0.500	0.125–0.250	0.075–0.125	0.050–0.075	0.025–0.050	0.010–0.025
Γ	1.75–2.00	**11.36%**	**13.93%**	**3.35%**	**1.00%**	0.53%	0.24%
	1.50–1.75	**5.56%**	**13.70%**	**5.99%**	**2.34%**	**1.70%**	0.67%
	1.25–1.50	**1.80%**	**7.76%**	**5.61%**	**3.11%**	**2.82%**	**1.56%**
	1.00–1.25	0.38%	**2.71%**	**2.87%**	**2.26%**	**2.33%**	**1.58%**
	0.75–1.00	0.07%	0.54%	**0.82%**	0.75%	**1.00%**	0.81%
	0.50–0.75	0.01%	0.09%	0.15%	0.18%	0.23%	0.17%

C_S and C_H. The top right panel of Figure 2 shows the joint posterior draws of C_S and C_H resulting from the Gibbs sampler; a large dot in the diagram represents the true values of the X-ray colors. In the bottom row of Figure 2 presents the three-dimensional histogram of the draws to the left and the contour plot to the right.

Because the Monte Carlo draws are superimposed on the grids of the power-law and thermal models in the color-color diagram, we can reversely infer the parameters of the models by computing posterior probabilities corresponding to each section split by the grids. Table 1 presents the normalized posterior probabilities of the X-ray colors in the grid of the power-law model. The 95% highest joint posterior density (HJPD) region is shown in bold face. If the power-law model is believed for this source, the most likely parameter values are $\widehat{N}_H = (0.125 - 0.250)$ and $\widehat{\Gamma} = (1.75 - 2.00)$.

2.2 Cluster Analysis for Galaxy Sources

With a survey of X-ray sources, hardness ratios can be used to answer scientific questions of interest. For example, the negative relationship between the soft band X-ray flux (λ_S) and the reciprocal of the simple hardness ratio ($1/\mathrm{R} = \lambda_H/\lambda_S$) is of interest; in this case, the energy spectrum is divided into two sub-energy bands. This scientific question specifically means that sources with fewer soft counts tend to have more hard counts per unit soft count. Brandt *et al.* (2001) report this negative relationship on a log scale, based on the method of moments. However, the correlation between $\log_{10}\lambda_S$ and $\log_{10}(\lambda_H/\lambda_S)$ is analytically decomposed into

$$\mathrm{Corr}\left(\log_{10}\lambda_S, \log_{10}\frac{\lambda_H}{\lambda_S}\right) = \frac{\mathrm{Corr}(\log_{10}\lambda_S, \log_{10}\lambda_H)\frac{\sqrt{\mathrm{Var}(\log_{10}\lambda_H)}}{\sqrt{\mathrm{Var}(\log_{10}\lambda_S)}} - 1}{\sqrt{\mathrm{Var}(\log_{10}\lambda_H - \log_{10}\lambda_S)}/\sqrt{\mathrm{Var}(\log_{10}\lambda_S)}}, \quad (14)$$

and its sign is negative if and only if the numerator is less than zero. In other words, the correlation of scientific interest becomes negative when the slope for regressing $\log_{10}\lambda_H$ on $\log_{10}\lambda_S$ is less than one, i.e.,

$$\varphi \equiv \mathrm{Corr}(\log_{10}\lambda_S, \log_{10}\lambda_H)\frac{\sqrt{\mathrm{Var}(\log_{10}\lambda_H)}}{\sqrt{\mathrm{Var}(\log_{10}\lambda_S)}} < 1. \quad (15)$$

Thus, the scientific question must be re-formalized in terms of the regression slope φ. If the regression slope is zero, knowing $\log_{10}\lambda_S$ does not help explain the variation in $\log_{10}\lambda_H$. However, a zero regression slope results in a negative overall correlation in (14), thereby misleading its interpretation.