Quantifying the Non-Existent

Multi-wavelength Model Fitting with Non-Detects

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Outline

- Context – Gamma Ray Bursts and their Afterglows
  - The Phenomenon
  - Light Curves and Modeling
- Radio Interferometry
  - Visibilities and The van Cittert – Zernike Theorem
  - Deconvolution
  - The Measurement Process
- Incorporating Non-detects
Gamma Ray Bursts

\[ \Gamma \sim 10^2 - 10^3 \quad \Gamma \sim 10 \]
GRB Lightcurves

- Light curve = $F_\nu(t)$
- Afterglow first appears in X-rays
- Peak emission moves to lower frequencies with time

$Laskar et al. (in prep.)$

$M(\vec{P}) = F_M(\nu, t)$
Multi-wavelength modeling

\[ M(\vec{P}) = F_M(\nu, t) \]

Laskar et al. (in prep.)

Frail et al. (2006)
Radio Interferometry

\[ V(u, v) = \lim_{T \to \infty} \frac{1}{T} \int E_1(t) E_2(t) \, dt \]

\[ V(u, v) = \int I(\alpha, \beta) e^{2\pi i (u\alpha + v\beta)} \, d\alpha \, d\beta \]
Measuring $V(u,v)$

\[ V_m(u,v) = W(u,v)V(u,v) \]
\[ \text{IFT}(V_m(u,v)) = \text{IFT}(W) \ast \text{IFT}(V) = B(\alpha, \beta) \ast I(\alpha, \beta) \]
Deconvolution

The CLEAN Algorithm:

1. "Dirty image" = IFT V
2. Clean: obtain clean components and residuals
3. Restore: convolve cc with "clean beam" and add back residuals

Find map max
Subtract (gain x dirty beam at position of peak) from map
Add δ-function to clean components list
Repeat

Hogbom (1974)
Detection?

- Fit the source with the known clean beam (elliptical Gaussian)
- Obtain source parameters (shape, size), both convolved with and deconvolved from clean beam
- Non-detect? Report upper limit = 3 x rms of map …
Model Fitting – Maximum Likelihood

\[ L = \prod_{i=1}^{N} P[t_i, \delta_i] = \prod_{i=1}^{N} [f(t_i)]^{\delta_i} [1 - S(t_i)]^{1-\delta_i} \]

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

\[ S(x) = 1 - \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x-\mu}{\sqrt{2\sigma}} \right) \right] \]

\[ t_i = \min(x_i, c_i) \]

\[ \delta_i = \begin{cases} 0 & \text{(censored)} \\ 1 & \text{(detected)} \end{cases} \]

\( x_i \) : detected values

\( c_i \) : upper limits