Incorporating Uncertainties in Atomic Data Into the Analysis of Solar Observations

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Acknowledgments

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"Improving the Analysis of Solar and Stellar Observations"

- The solar corona is a complex and dynamic system
- Measuring physical properties in any solar region is important for understanding the processes that lead to these events

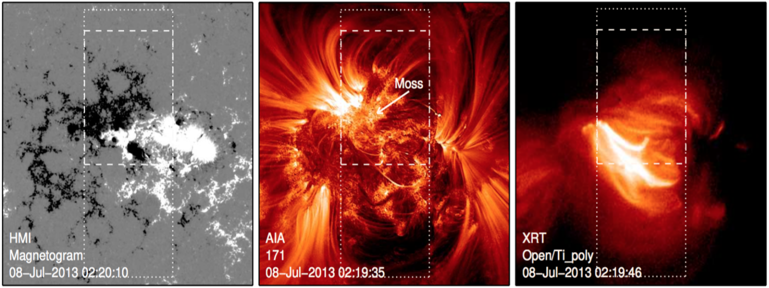


Figure: The photospheric magnetic field measured with HMI, million degree emission observed with the AIA Fe IX 171,Å channel, and high temperature loops observed with XRT

The Problem

- We want to infer physical quantities of the solar atmosphere (density, temperature, path length, etc.), but we only observe intensity
- Inferences rely on models for the underlying atomic physics
- How to address **uncertainty** in the atomic physics models?

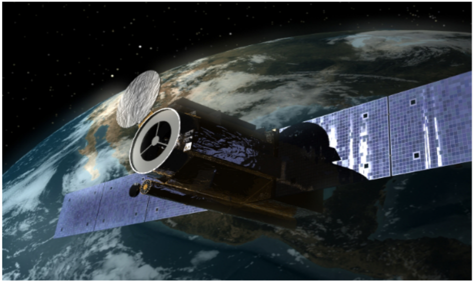


Figure: Hinode spacecraft. Image credit: NASA/GSFC/C. Meaney

Physical Parameters

- k: pixel index, of 1000 randomly selected from the image
- n_{ek}: number of free electrons per unit volume in plasma
- T_{ek}: electron temperature
- $\mathrm{d}s_k$: path length through the solar atmosphere
- $\theta_k = (\log \mathrm{n}_{ek}, \log \mathrm{d}s_k)$
- m: index of the emissivity curve
- Expected intensity of line with wavelength λ:

$$\epsilon_\lambda^{(m)}(\mathrm{n}_{ek}, \mathrm{T}_{ek})\mathrm{n}_{ek}^2 \mathrm{d}s_k$$

- $\epsilon_\lambda^{(m)}(\mathrm{n}_{ek}, \mathrm{T}_{ek})$ is the plasma emissivity for the line with wavelength λ in pixel k

Data: Observed Intensity

- Data from the Extreme-Ultraviolet Imaging Spectrometer (EIS) on *Hinode* spacecraft.

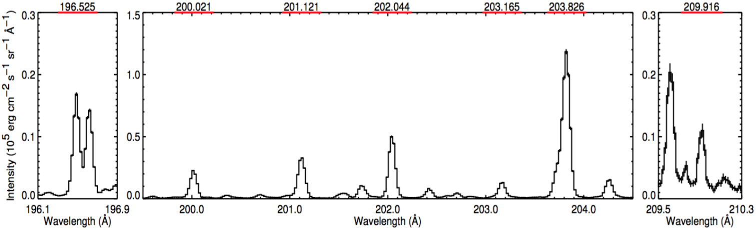


Figure: Example EIS spectrum of seven Fe XIII lines

- Spectral lines with wavelengths $\Lambda = \{\lambda_1, \ldots, \lambda_J\}$
- Observed intensities for K pixels and J wavelengths:

$$\hat{D} = \{D_k = (I_{k\lambda_1}, \ldots, I_{k\lambda_J}), k = 1, \ldots, K\}$$

- Standard deviation $\sigma_{k\lambda_j}$ are also measured

Uncertainty: Emissivity

- Emissivity: how strongly energy is radiated at a given wavelength
- Simulated from a model accounting for uncertainty in the atomic data
- Suppose a collection of M emissivity curves are known

$$\mathcal{M} = \{\epsilon_\lambda^{(m)}(\mathrm{n}_{ek}, \mathrm{T}_{ek}), \lambda \in \Lambda, m = 1, \ldots, M\}$$

- $m = 1$: the **default** value from CHIANTI
- Treating all pixels independently, but with same emissivities

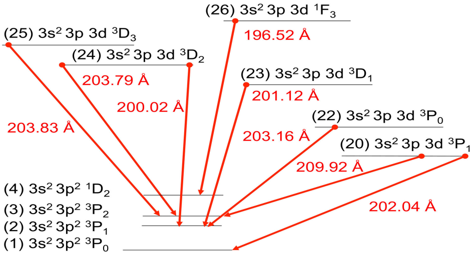


Figure: A simplified level diagram: transitions relevant to the 7 lines considered.

Aim

- **To infer density**
- In Fe XIII, density dependence of emissivities is **not sensitive** to temperature

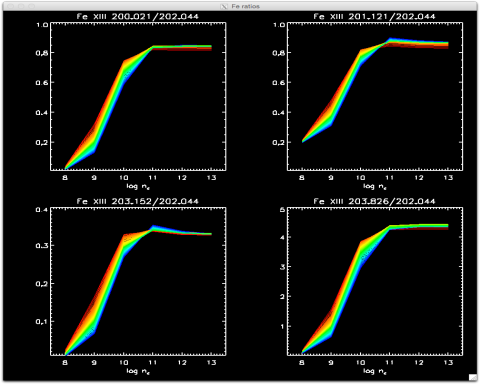


Figure: Relationship of emissivities and density when temperature changes in Fe XIII case.

Bayesian Inference

- **Prior distribution**: quantify the uncertainty in the values of the unknown model parameters **before** the data is observed
- **Likelihood**: the distribution of the data given the model parameters
- **Posterior distribution**: quantify the uncertainty in the values of the unknown model parameters **after** the data is observed
- Relationship:

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

Statistical Model

- Let $\mathcal{M} = \{\epsilon_\lambda^{(m)}(\mathrm{n}_{ek}, \mathrm{T}_{ek}), \lambda \in \Lambda, m = 1, \ldots, M\}$, emissivity curves
- Let $\hat{D} = \{D_k = (I_{k\lambda_1}, \ldots, I_{k\lambda_J}), k = 1, \ldots, K\}$, observed intensity

Likeihood $p(D_k \mid m, \theta_k)$

$$I_{k\lambda} \mid m, \mathrm{n}_{ek}, \mathrm{d}s_k \overset{\text{indep}}{\sim} \text{Normal}\left(\epsilon_\lambda^{(m)}(\mathrm{n}_{ek}, \mathrm{T}_{ek})\mathrm{n}_{ek}^2 \mathrm{d}s_k,\ \sigma_{k\lambda}^2\right), \quad \text{for } \lambda \in \Lambda \quad (1)$$

Prior Distributions

- **Independent** prior distributions:

$$p(m, \theta_k) = p(m)\ p(\log \mathrm{n}_{ek})\ p(\log \mathrm{d}s_k) \tag{2}$$

Prior distributions of $p(m)$ and $p(\theta_k)$:

$$m \sim \mathrm{DiscreteUniform}(\{1, \ldots, M\}) \tag{3}$$

$$\log_{10} \mathrm{n}_{ek} \sim \mathrm{Uniform}(\min = 7, \max = 12) \tag{4}$$

$$\log_{10} \mathrm{d}s_k \sim \mathrm{Cauchy}(\mathrm{center} = 9, \mathrm{scale} = 5) \tag{5}$$

- Note: a flat prior, $p(\log \mathrm{d}s_k) \propto 1$, yields an improper posterior distribution because likelihood $\to$ constant > 0 as $\log \mathrm{d}s_k \to -\infty$
- Here we use a sample of $M = 1000$ emissivity curves

Posterior Distribution and Methods

- Use **Bayesian Methods** to incorporate information in the data for narrowing the uncertainty in the atomic physics calculation

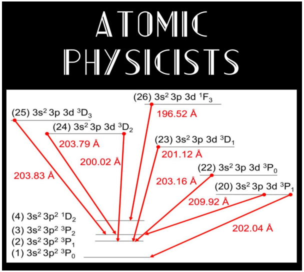
Joint posterior distribution of m and $\theta_k = (\log \mathrm{n}_{ek}, \log \mathrm{ds}_k)$

$$p(m, \theta_k \mid D_k) \propto p(D_k \mid m, \theta_k)\ p(m, \theta_k), \quad (6)$$

- m is treated as an **unknown parameter**
- Have specified the **likelihood** and **prior** distribution

- **Aim**: To obtain a Monte Carlo (MC) sample from $p(m, \theta_k \mid D_k)$
- Strategy: two-step Monte Carlo samplers
- **Separate and joint analysis**: Deal with pixels individually or jointly

Using a sample for $p(m)$ makes it easy for different team experts to work in parallel:



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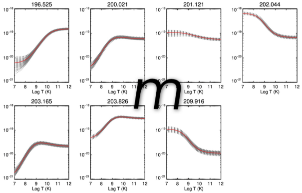

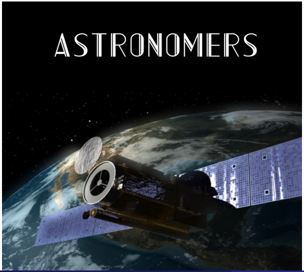

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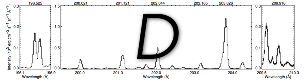


Pragmatic vs. Fully Bayesian Methods

- Joint posterior distribution:

$$p(m, \theta_k \mid D_k) = \frac{p(D_k \mid m, \theta_k) p(m, \theta_k)}{p(D_k)} \tag{7}$$

- How to handle uncertainty in ?

Pragmatic Bayesian

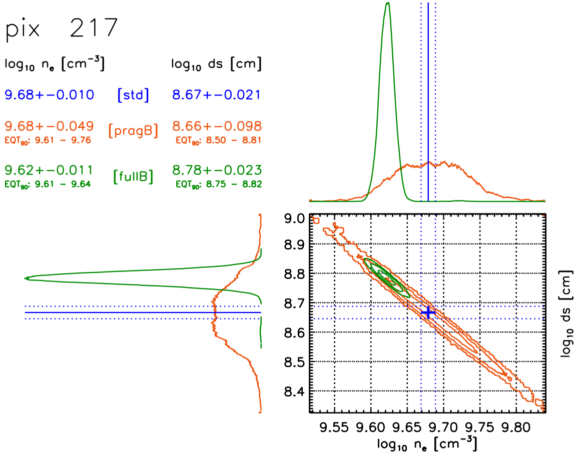
$$p(m, \theta_k \mid D_k) = p(\theta_k \mid D_k, m)\ p(m). \tag{8}$$

- $M = 1000$ **equally likely** emissivity curves as a priori

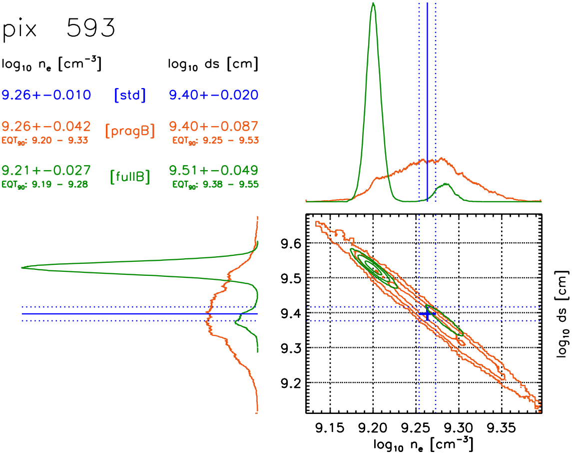
Fully Bayesian

$$p(m, \theta_k \mid D_k) = p(\theta_k \mid D_k, m)\ p(m \mid D_k). \tag{9}$$

Compare inferred parameters via diff methods for pixel 217

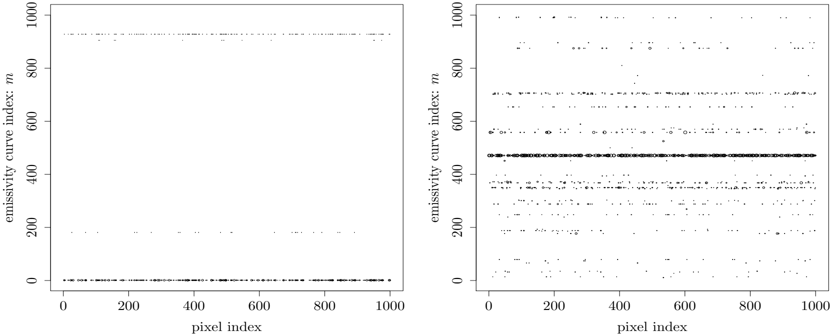


Compare inferred parameters via diff methods for pixel 593



Multimodal Posterior Distributions

- **Bimodal** posterior distributions occur
  - **Two modes** correspond to **two emissivity curves**, Emis_{471} and Emis_{368}
- Reason: **Not enough** emissivity curves
- Challenge: **Sparse** selection of emissivity curves



Compare selected emissivities with default

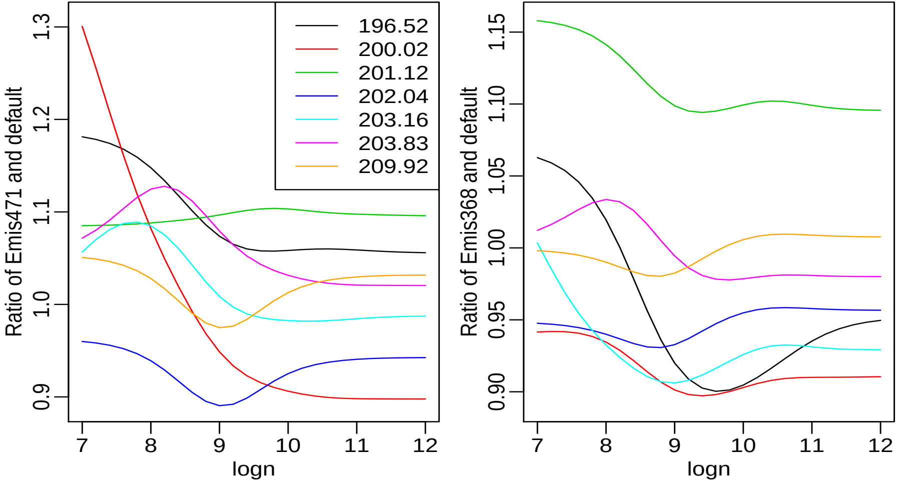


Figure: Plot of ratio of selected emissivities and default CHIANTI over 7 lines.

Conclusions

- Use a **Bayesian framework** to interpret the observed intensities in the context of the different realizations of the atomic data
- A **pragmatic Bayesian** approach, where each realization of emissivities is considered to be **equally likely**, yields uncertainties in the electron density and path length that are **larger** than the statistical uncertainty implied by fluctuation in data alone
- A **fully Bayesian** approach, where we **allow the observed intensities to update the uncertainty** in the emissivity curves, **reduces the uncertainties** in the plasma parameters
- A **different realization** of the atomic data is **more likely** than the default CHIANTI calculation

Nest Step

- Come up with a way to efficiently represent **the high dimensional joint distribution** of the uncertainty of the emissivity curves
- An algorithm: summarizing the distribution with multivariate (standard) Normal distribution via **principal component analysis (PCA)**