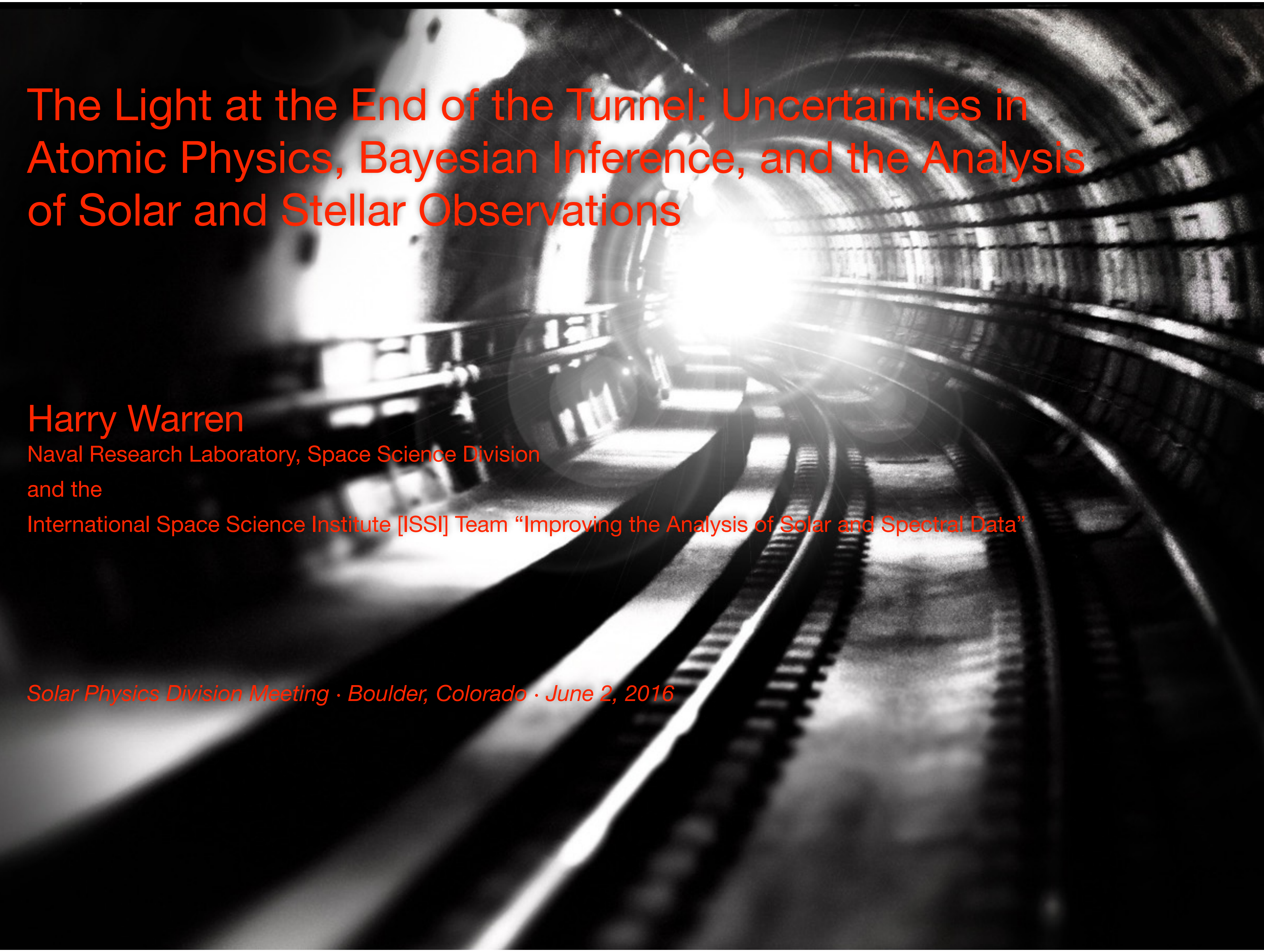

The Light at the End of the Tunnel: Uncertainties in Atomic Physics, Bayesian Inference, and the Analysis of Solar and Stellar Observations

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and the

International Space Science Institute [ISSI] Team “Improving the Analysis of Solar and Spectral Data”

Solar Physics Division Meeting · Boulder, Colorado · June 2, 2016

ISSI Team: Improving the Analysis of Solar and Stellar Observations

Team members are

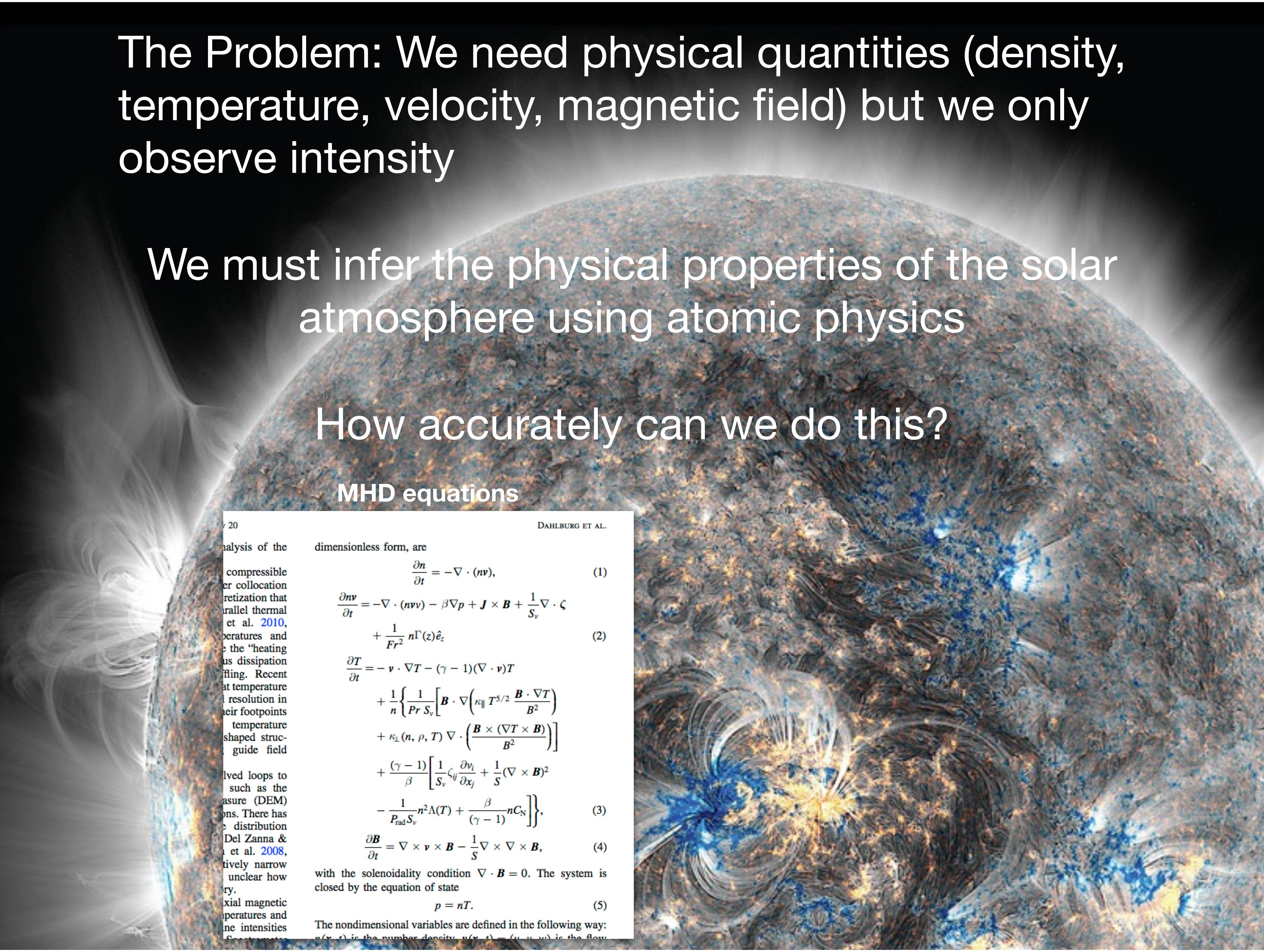
- Harry Warren (NRL; PI)
- Chloe Guennou (Inst. Astr. Spatiale)
- David Stenning (UCI)
- David van Dyk (Imperial)
- Fabio Reale (Palermo)
- Frederic Auchere (Inst. Astr. Spatiale)
- Giulio del Zanna (Cambridge)
- Inigo Arregui (Inst. Astr. de Canarias)
- Jessi Cisweski (CMU)
- Mark Weber (SAO)
- Nathan Stein (UPenn)
- Randall Smith (SAO)
- Vinay Kashyap (SAO)
- Adam Foster (SAO)
- Connor Ballance (Queen's University Belfast)

Special Thanks to the always helpful Peter Young!

The Problem: We need physical quantities (density, temperature, velocity, magnetic field) but we only observe intensity

We must infer the physical properties of the solar atmosphere using atomic physics

How accurately can we do this?



MHD equations

dimensionless form, are

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\boldsymbol{v}), \quad (1)$$

$$\frac{\partial n\boldsymbol{v}}{\partial t} = -\nabla \cdot (n\boldsymbol{v}\boldsymbol{v}) - \beta \nabla p + \boldsymbol{J} \times \boldsymbol{B} + \frac{1}{S_v} \nabla \cdot \zeta + \frac{1}{Fr^2} n\Gamma(z)\hat{e}_z \quad (2)$$

$$\frac{\partial T}{\partial t} = -\boldsymbol{v} \cdot \nabla T - (\gamma - 1)(\nabla \cdot \boldsymbol{v})T + \frac{1}{n}\left\{ \frac{1}{Pr\, S_v}\left[\boldsymbol{B} \cdot \nabla \left(\kappa_\parallel T^{5/2} \frac{\boldsymbol{B} \cdot \nabla T}{B^2} \right) + \kappa_\perp(n, \rho, T)\, \nabla \cdot \left(\frac{\boldsymbol{B} \times (\nabla T \times \boldsymbol{B})}{B^2} \right) \right] + \frac{(\gamma - 1)}{\beta}\left[\frac{1}{S_v} \zeta_{ij} \frac{\partial v_i}{\partial x_j} + \frac{1}{S} (\nabla \times \boldsymbol{B})^2 - \frac{1}{P_{\rm rad} S_v} n^2 \Lambda(T) + \frac{\beta}{(\gamma - 1)} n C_N \right] \right\}, \quad (3)$$

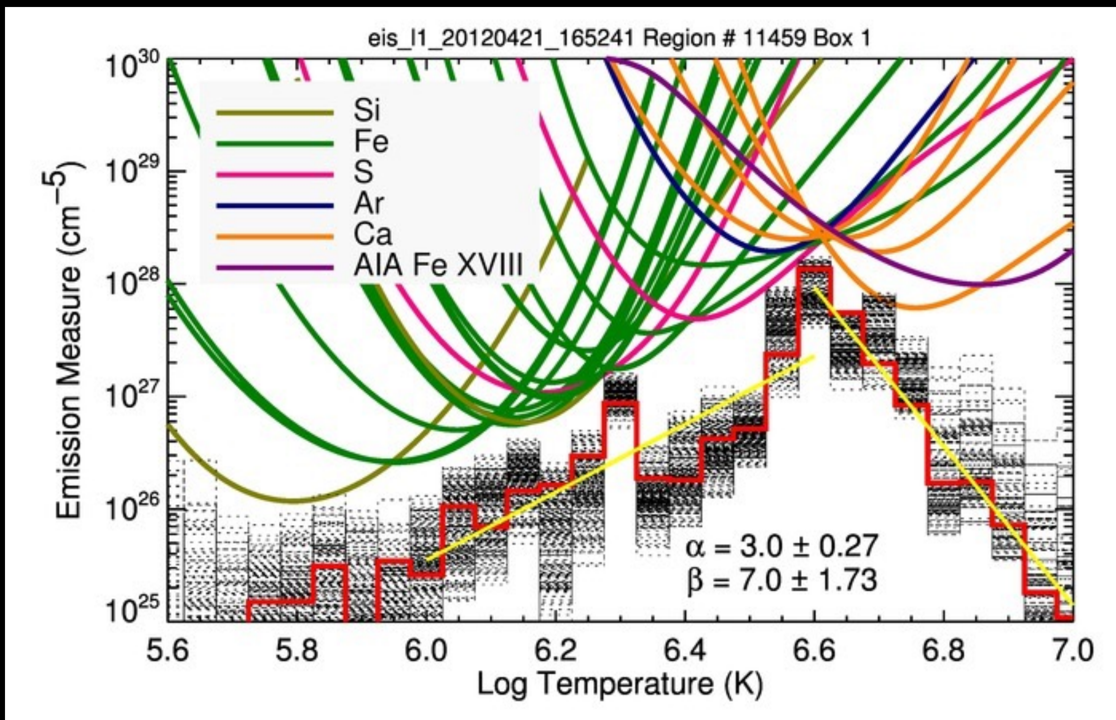
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{v} \times \boldsymbol{B} - \frac{1}{S} \nabla \times \nabla \times \boldsymbol{B}, \quad (4)$$

with the solenoidality condition $\nabla \cdot \boldsymbol{B} = 0$. The system is closed by the equation of state

$$p = nT. \quad (5)$$

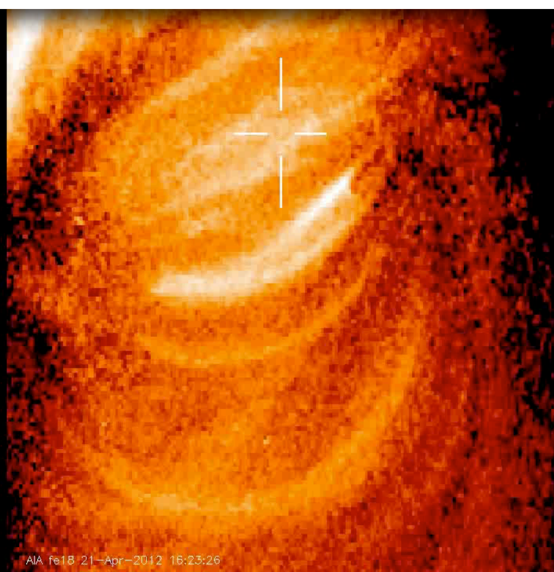
The nondimensional variables are defined in the following way:

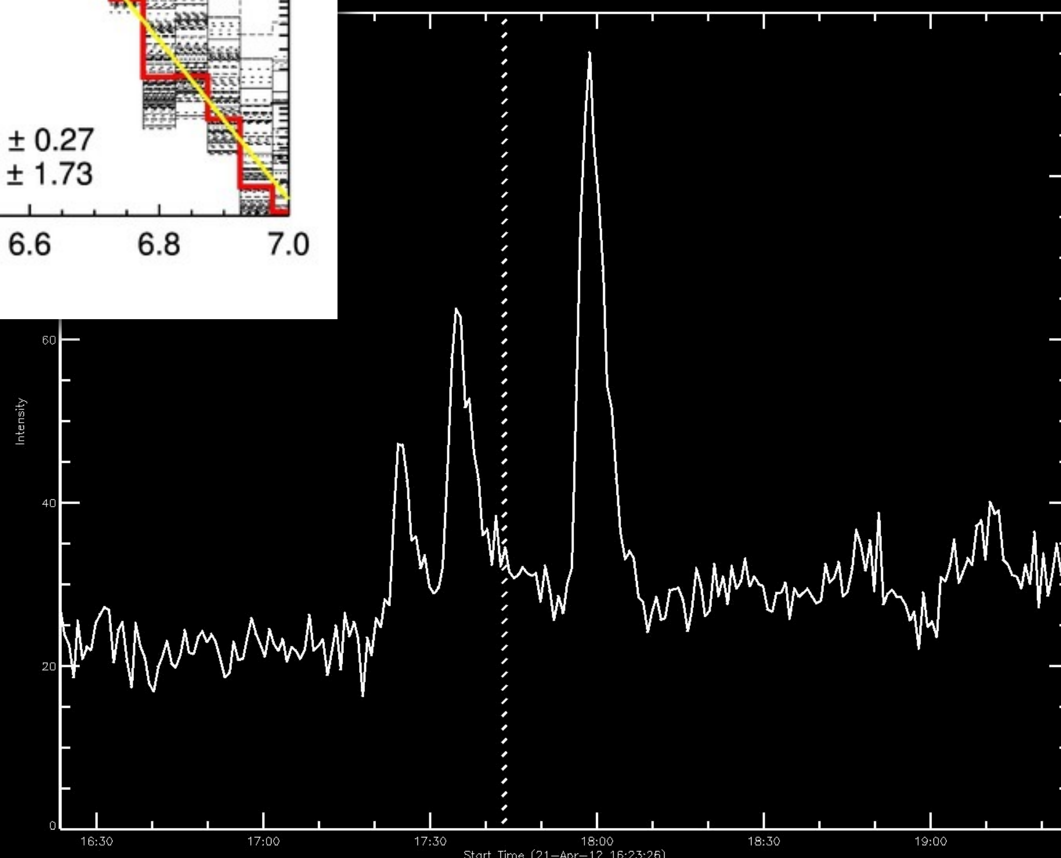
Example: We want to use the DEM to constrain models of coronal heating, but how well can it be computed? Errors in the atomic data are currently unaccounted for and likely dominate the calculation

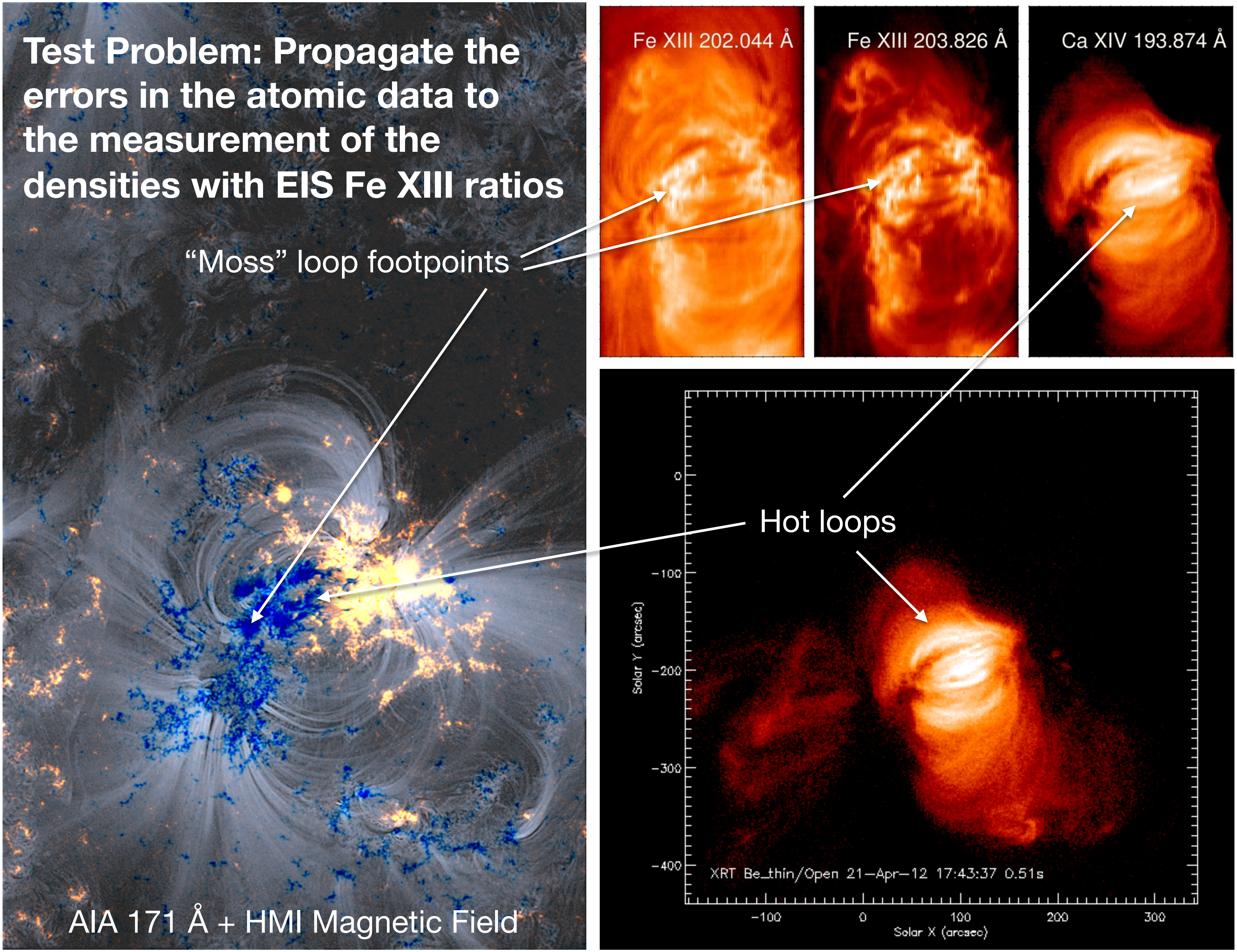


For example, see Guennou et al. 2013
http://adsabs.harvard.edu/abs/2013ApJ...774...31G

AIA Fe XVIII

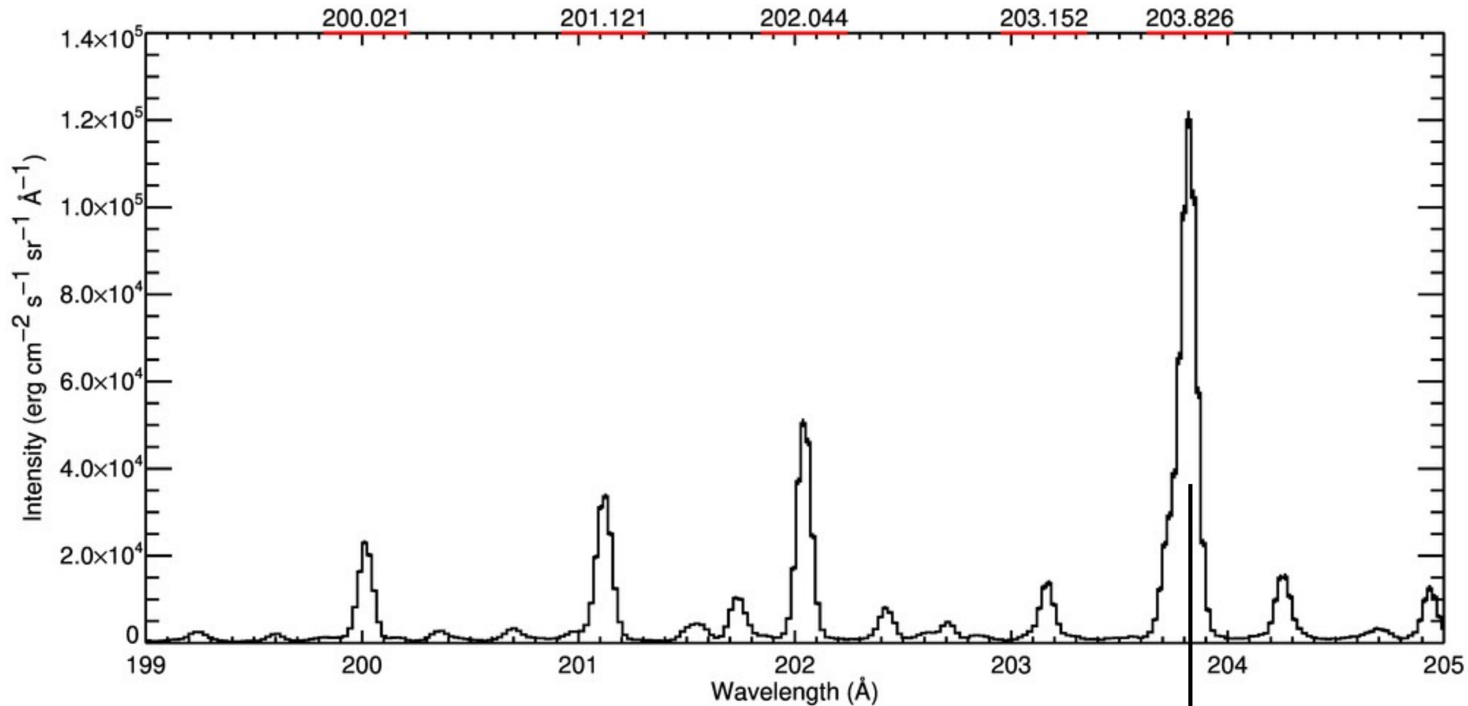


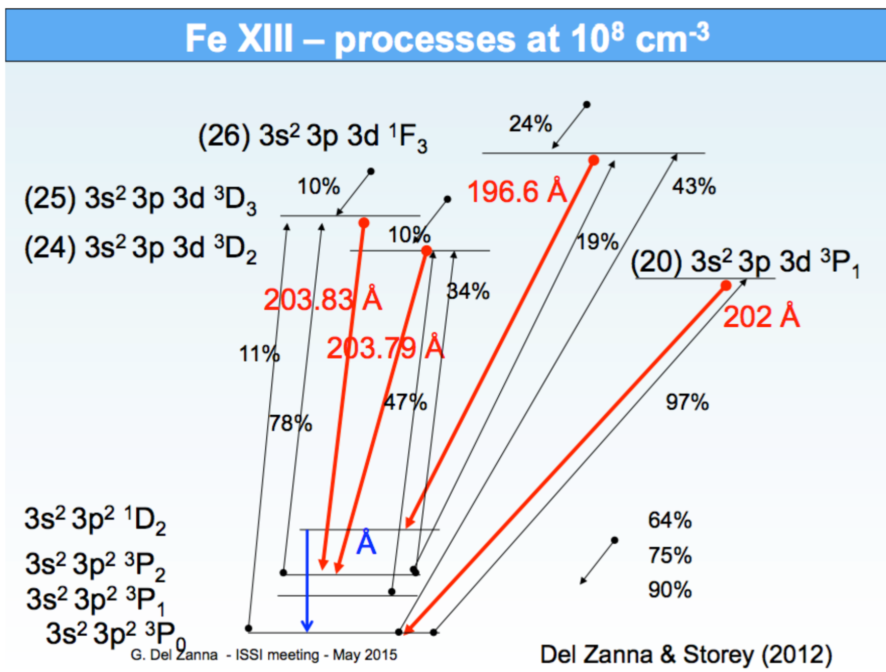




Test Problem: Propagate the errors in the atomic data to the measurement of the densities with EIS Fe XIII ratios
"Moss" loop footpoints
AIA 171 Å + HMI Magnetic Field
Fe XIII 202.044 Å
Fe XIII 203.826 Å
Ca XIV 193.874 Å
Hot loops
XRT Be_thin/Open 21-Apr-12 17:43:37 0.51s
Solar Y (arcsec)
Solar X (arcsec)

Example EIS Spectrum Near 200 Å: 5 Fe XIII Lines





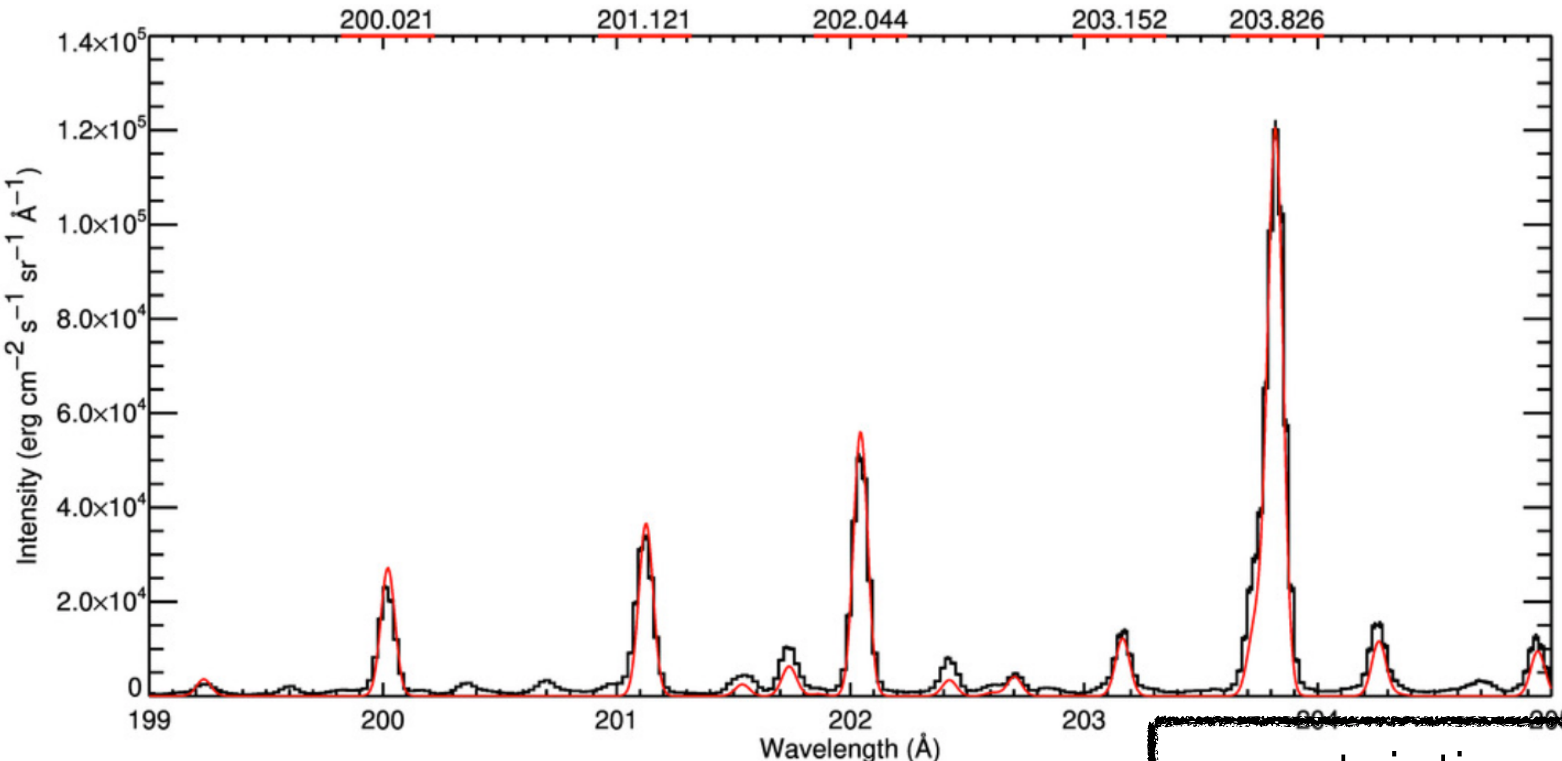
intensity

$$I_\lambda = \epsilon_\lambda(n_e, T_e)\, n_e^2\, ds$$

atomic data

electron density, temperature

Least-Squares Fit to the Intensities



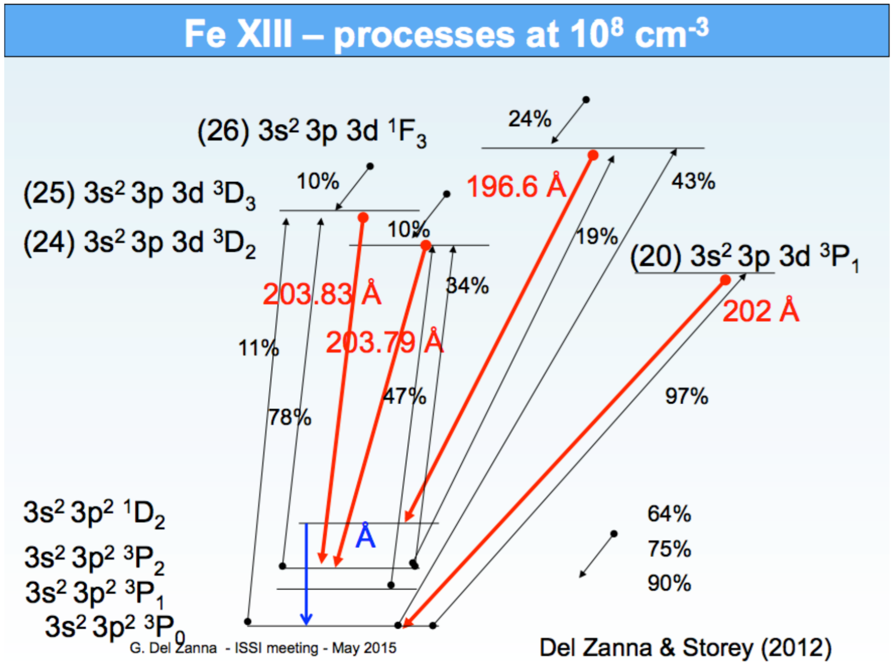
uncertainties appear to be very small!

$$I_\lambda = \epsilon_\lambda(n_e, T_e)\, n_e^2\, ds$$

```
 model log_n = 9.48 +- 0.016
model log_ds = 9.14 +- 0.031
        chi2 = 153.2
```

Line	Imodel	Iobs	SigmaI	dI/Sigma	dI/I
200.021	2064.02	1809.67	32.90	7.73	14.1
201.121	2506.85	2946.72	51.14	8.60	14.9
202.044	4299.64	4153.84	64.85	2.25	3.5
203.152	932.87	1071.74	48.20	2.88	13.0
203.826	10223.57	10620.57	160.95	2.47	3.7

Phase 1: Perturb CHIANTI collision strengths and decay rates uniformly, generate 1000 realizations of the atomic data




```
CHIANTI: setup_ion.pro

rnd = randomn(seed, splstr.info.ntrans)

for i=0,splstr.info.ntrans-1 do begin

  perturb = 1 + CHIANTI_PERTURB*rnd[i]

  ;; --- perturb; spl is -1 for missing data, so don't overwrite
  good = where(splstr.data[i].spl ge 0, n_good)
  if n_good gt 0 then splstr.data[i].spl[good] = splstr.data[i].spl[good]*perturb

endfor

CHIANTI: read_wgda.pro

perturb = 1 + CHIANTI_PERTURB_AVAL*randomn(seed, nindex)
a_value = a_value*perturb

For simplicity only save emissivities for 5 lines of interest

Each run takes about 10s
```

Re-Analyze the Data

Original CHIANTI

model log_n = 9.48 +- 0.016
model log_ds = 9.14 +- 0.031
chi2 = 153.2

Line	Imodel	Iobs	SigmaI	dI/Sigma	dI/I
200.021	2064.02	1809.67	32.90	7.73	14.1
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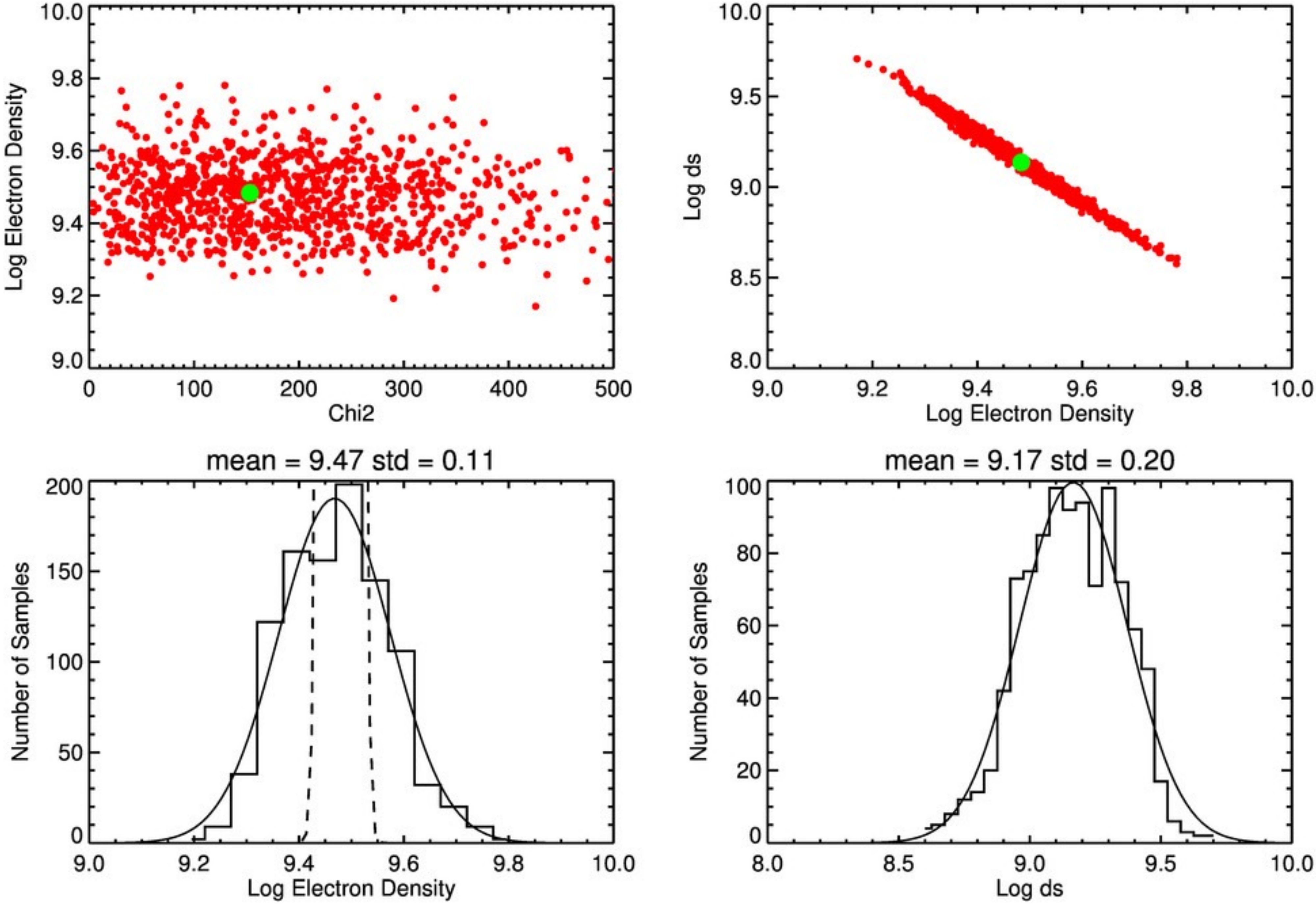
203.826	10223.57	10620.57	160.95	2.47	3.7

Example with 10% perturbation

model log_n = 9.80 +- 0.026
model log_ds = 8.53 +- 0.050
chi2 = 68.5

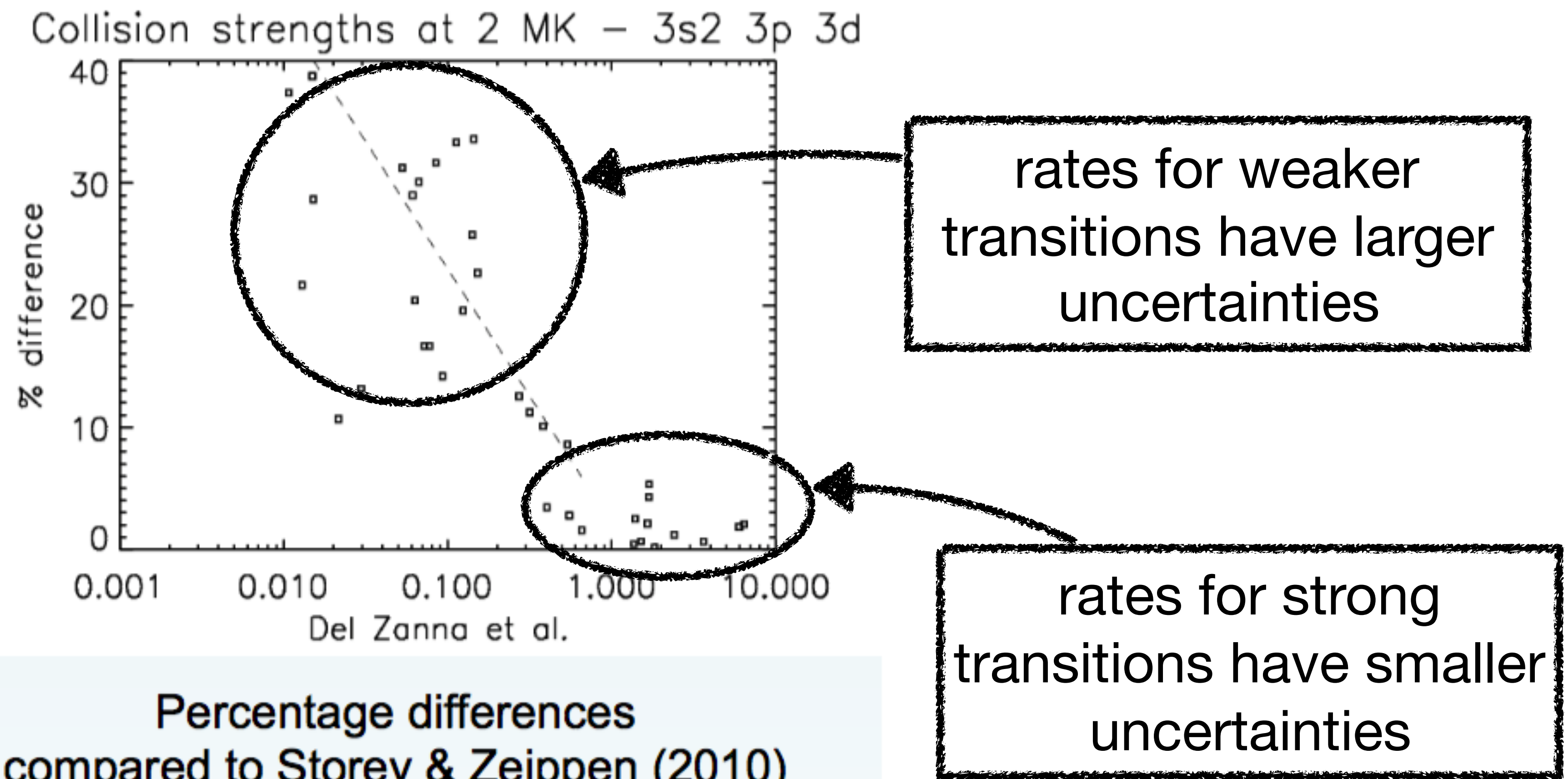
Line	Imodel	Iobs	SigmaI	dI/Sigma	dI/I
200.021	1775.97	1809.67	32.90	1.02	1.9
201.121	2608.44	2946.72	51.14	6.62	11.5
202.044	4283.36	4153.84	64.85	2.00	3.1
203.152	997.34	1071.74	48.20	1.54	6.9
203.826	11290.98	10620.57	160.95	4.17	6.3

Results From 1000 Runs at 10%: Errors in the atomic data dominate



But a uniform 10% is not right! . . . the rates for some transitions are better known that this, some are much more uncertain

Giulio Del Zanna compared rates from recent calculations and estimated the uncertainty as a function of collision strength



Percentage differences compared to Storey & Zeippen (2010) at peak Te=2 MK.

Phase 2: Perturb CHIANTI collision strengths and decay rates with GDZ method, generate 1000 realizations of the atomic data

How do we "learn" from the data? . . . pragmatic and full Bayes

[Nathan Stein, David Stenning, David van Dyke, Jessi Cisweski]

assume that the observations and the atomic data are independent

- **Pragmatic Bayes:** Assume that $P(A \mid Y) = P(A)$ such that

$$P(\Theta, A \mid Y) = P(\Theta \mid A, Y)\, P(A)$$

  - For $l = 1, \ldots, L$:

    Step 1: Sample A_l from its prior distribution: $A_l \sim P(A)$.

    Step 2: For $k = 1, \ldots, K$: sample $\Theta^{(k)}$ from its joint posterior distribution given A_l :

$$\Theta^{(k)} \sim P(\Theta \mid Y, A_l).$$

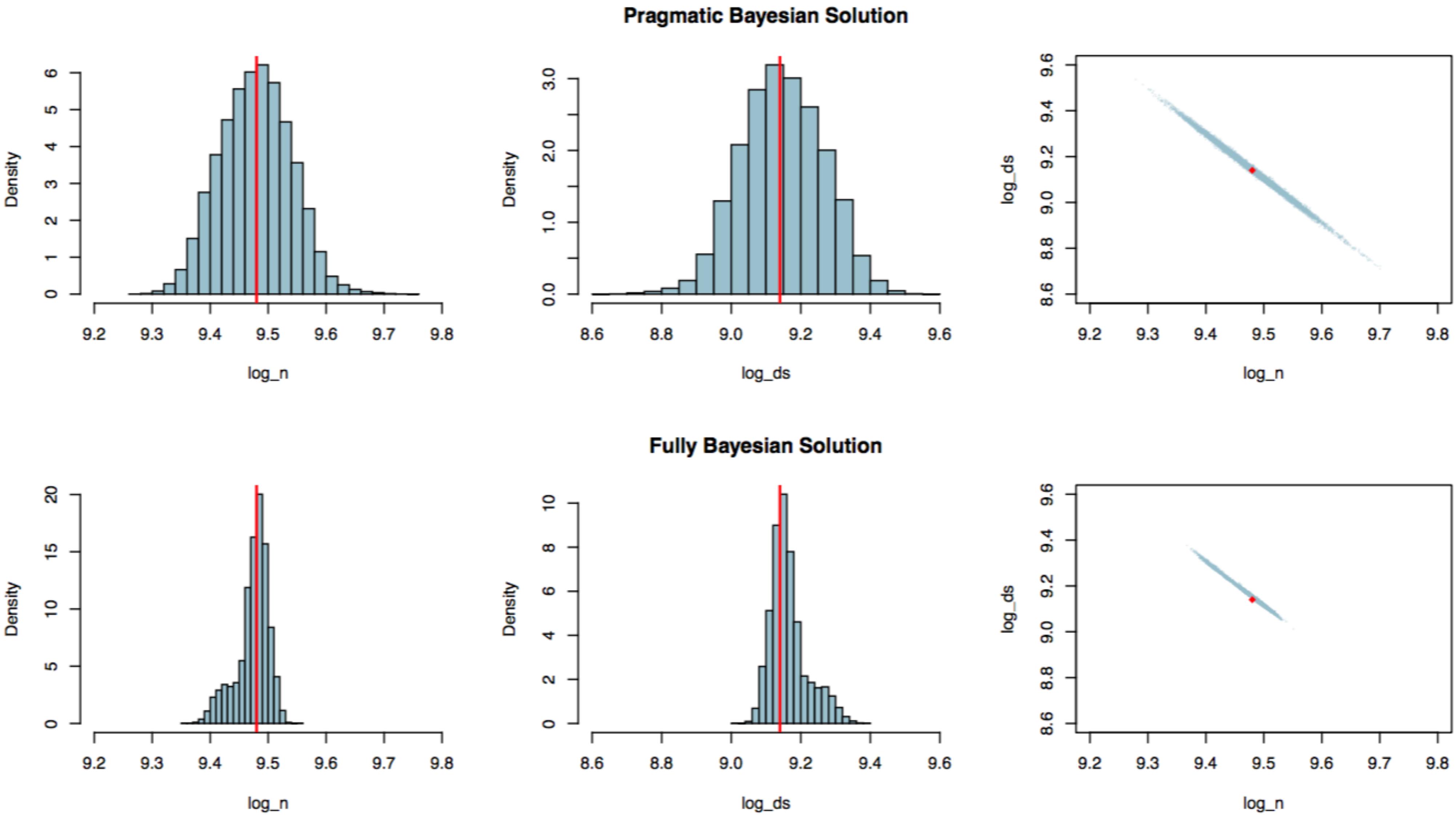
assume that the observations are conditional on the atomic data

- **Full Bayes:** $P(\Theta, A \mid Y) = P(\Theta \mid A, Y)\, P(A \mid Y)$.
- The fully Bayesian posterior distribution on Θ can be thought of as a weighted version of that under Pragmatic Bayes:

$$P(\Theta \mid Y) = \int P(\Theta \mid A, Y)\, \frac{P(A \mid Y)}{P(A)} P(A)\, \mathrm{d}A.$$

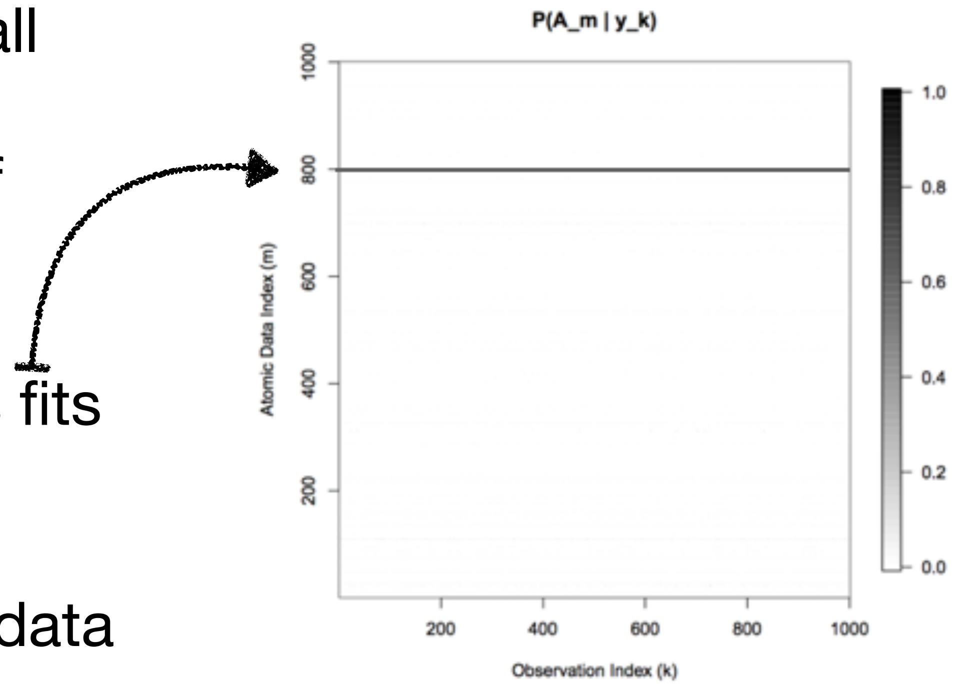
- Alternative computational techniques are used to obtain samples from $P(\Theta \mid Y)$.

Pragmatic vs Full Bayes



This is only applied to a single set of intensities

Analysis of full data set: consider all combinations of 1000 sets of EIS intensities and 1000 realizations of CHIANTI



Result: another set of atomic rates fits the data much better

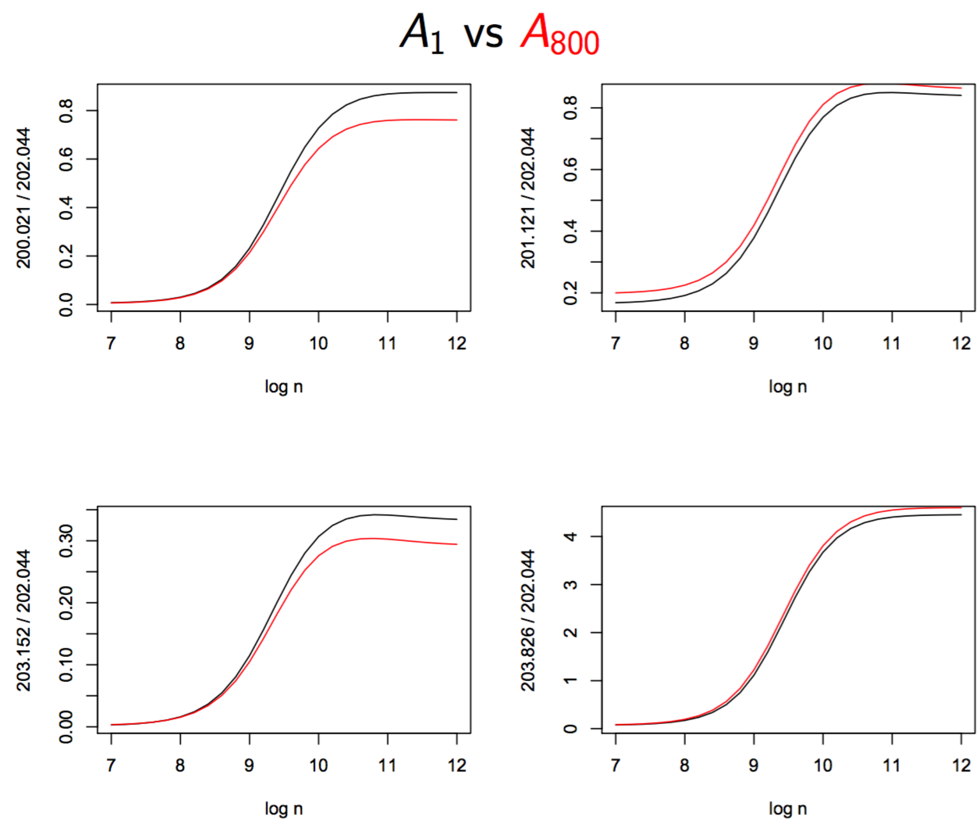
A clue that EIS analysis or atomic data can be improved

- Use Laplace's method to approximate

$$p(y_k|A_m) = \int p(y_k|\theta_k, A_m)\, p(\theta_k)\, d\theta_k$$

for the kth pixel and the mth atomic data curve

- From these, we can directly compute:
  - Separate analyses: $p(A_m|y_k) = p(y_k|A_m)/\sum_i p(y_k|A_i)$
  - Joint analysis: $p(A_m|y_1,\ldots,y_K) = \prod_k p(y_k|A_m)/\sum_i \prod_k p(y_k|A_i)$
- Use Gaussian approximations for posterior distributions $p(\theta_k|y_k, A_m)$
- Computing time for 1000 pixels: 6.5 hours



Summary and Conclusions

For the first time we have considered the analysis of spectroscopic data including both statistical errors and uncertainties in the atomic data

Uncertainties in the atomic data dominate, but are not catastrophic . . . phew!

More work is in progress

- Analysis of stellar O VII/O VIII spectra
- Analysis of collision rate covariance structure from first principle calculation
- Test problems using MHD simulation results